MATHEMATICS 9

Module 4: Algebra

Distance Learning

Alberta EDUCATION

Mathematics 9

Module 4

ALGEBRA

Distance Learning

Alberta EDUCATION

This document is intended for	
Students	✓
Teachers (Mathematics 9)	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 9
Student Module
Module 4
Algebra
Alberta Distance Learning Centre
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Welcome to Module 4!

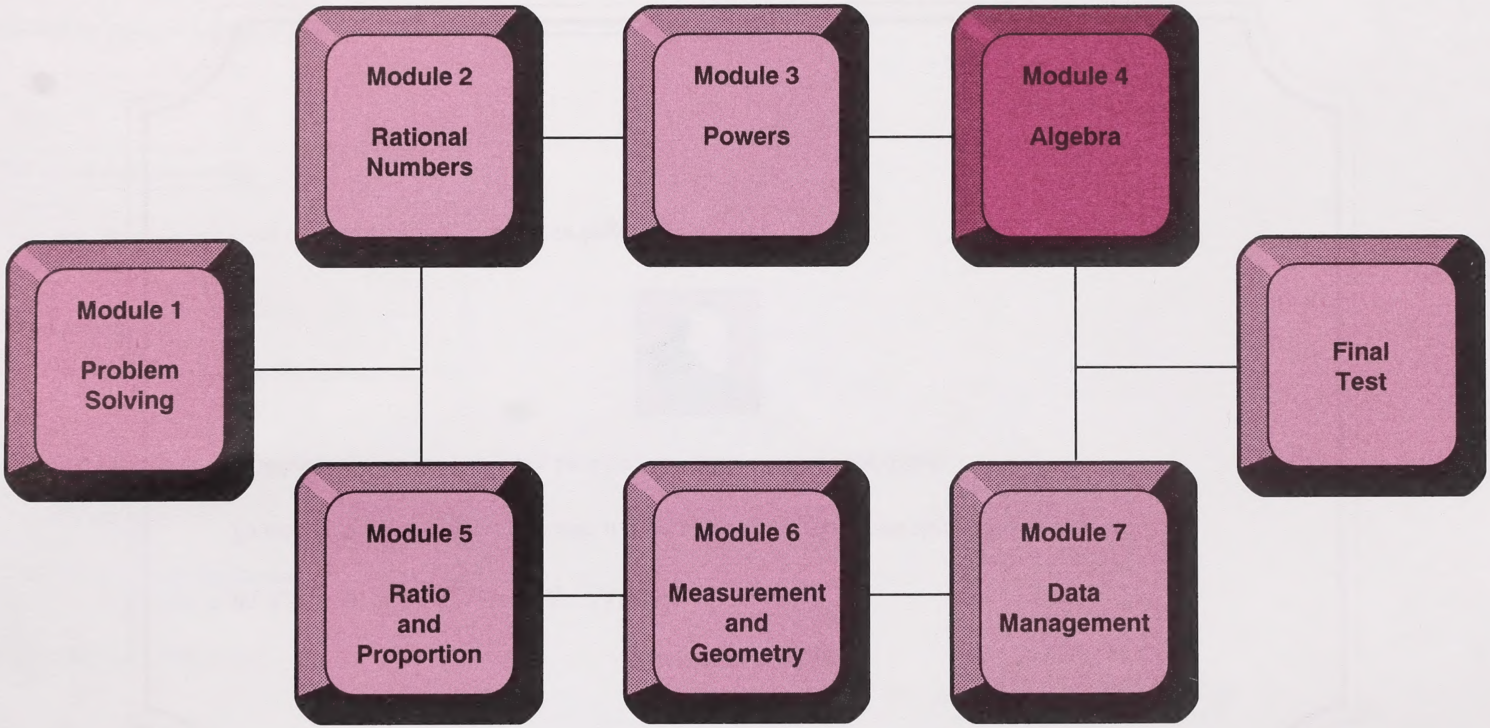
We hope you'll enjoy your study of ***Algebra****.*

To make your learning a bit easier, a teacher will help guide you through the material.

So whenever you see this icon, turn on your audiocassette and listen.

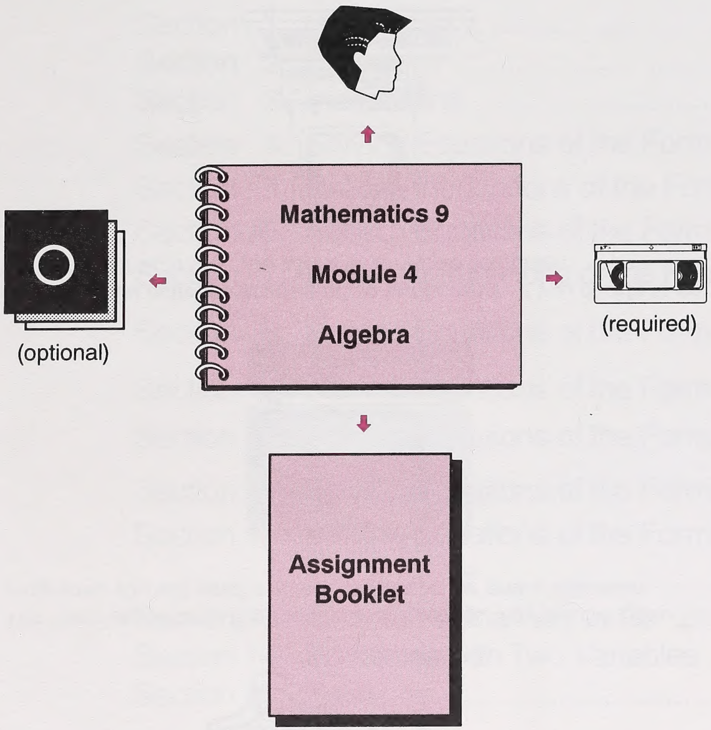
Turn the audiocassette on now to begin.

Course Overview



Mathematics 9 has seven modules and a final supervised test.

Module 4 Components



This booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module: the companion audiocassette, the videocassettes, the computer software, and the Assignment Booklet.

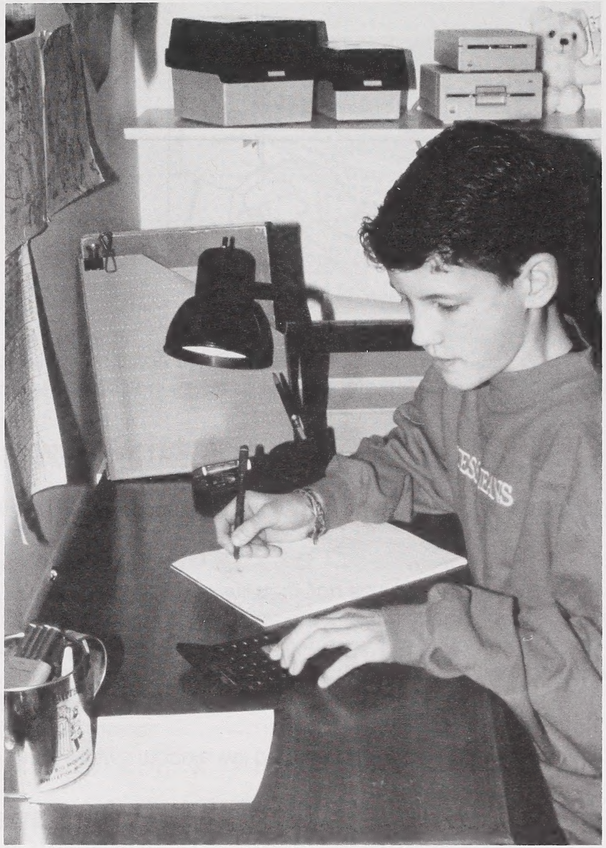
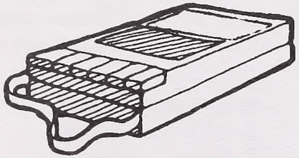


Photo Search Ltd.

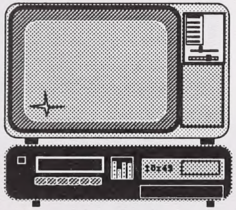
There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Optional Equipment

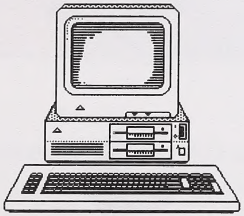
The companion audiocassette for this module is optional. If you decide to listen to it, you will need an audiocassette player.



The video activities in this module are required. To view the video programs, you will need a videocassette player and a television.



The computer activities in this module are optional. If you decide to do the computer activities, you will need an Apple computer.



Required Equipment

You will need a geometry set and a calculator for this module. The calculator should have an $a\frac{b}{c}$ key so that you can perform operations on fractions.

Evaluation

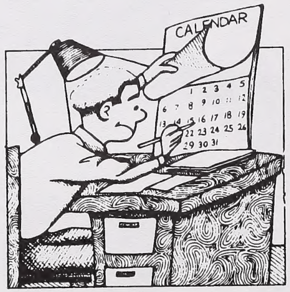
Your mark on this module will be determined by your work in the Assignment Booklet.

Your responses to the questions in this Student Module Booklet are not to be submitted for a grade. However, it is important that you work through the activities carefully before attempting the questions in the Assignment Booklet. This will help you achieve a greater degree of success in your studies.

Discuss how the module will be evaluated with your learning facilitator.

Time Management

Decide how long you will need to complete the module. Your learning facilitator will help you plan a schedule.



CONTENTS AT A GLANCE

MODULE INTRODUCTION

What Lies Ahead

In the module introduction you will preview Module 4.

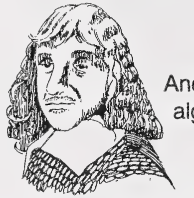
Working Together

In Module 4 of Mathematics 9 you will learn about **algebra**.

Algebra can be used to describe mathematical patterns, and it can also be used to solve problems.

You may be interested to learn that the study of algebra began in Baghdad around the eighth century. At that time the Arabian Empire stretched from Spain to India, and Baghdad was a cultural and scientific centre. Arabian scholars studied mathematics and developed algebra.

The English word *algebra* was borrowed from the Arabian word *al-jabr* which was used in the title of a book written by the Arabian mathematician, ibn-Musa Al-Khuwarizmi.



Another important person in the development of algebra was René Descartes (1596-1650).

Descartes was a French mathematician who developed the process of graphing algebraic relations using ordered pairs of numbers and describing these relations with equations.

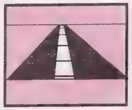
The story about how Descartes dicovered coordinate geometry is an interesting one.

As a boy Descartes had poor health, so the rector of the Jesuit school he attended often allowed him to stay in his room until he felt well enough to go to class.

During those mornings in his room, Descartes often amused himself by watching a fly on the ceiling and by trying to form an equation that described its path.

This practice led Descartes to develop the coordinate plane, which is sometimes called the **Cartesian plane** in recognition of his work.

GETTING SET



What Lies Ahead

This section will pretest the skills taught in this module.

Working Together

The Pretest in this section will help you and your learning facilitator to determine your strengths and weaknesses and individualize your learning plan.

Pretest

1. Solve each of the following equations by using inspection or the guess-check-revise method.

   a. $a+6=15$ b. $b-4=-4$ c. $5b=30$
   d. $3b=-24$ e. $4m+1=17$ f. $3s+5=2s-1$

2. Solve the following inequations by using inspection or the guess-check-revise method.

   a. $x+5>2$ b. $x-3<8$
   c. $3x<21$ d. $2x>16$

3. Solve each of the following equations using a systematic method. Show your work.

   a. $x+3=10$ b. $y-4=8$ c. $y-2=-5$
   d. $3a=18$ e. $-a=5$ f. $-2a=14$
   g. $\frac{x}{3}=8$ h. $\frac{x}{2}=\frac{3}{5}$ i. $3a+2a=15$
   j. $4a-a=18$ k. $2(m+5)=8$ l. $3(r-1)=6$
   m. $3a+2=5a$ n. $5a-1=2a+5$ o. $a^2+1=17$
   p. $3b^2=12$ q. $\sqrt{m}-1=3$ r. $2\sqrt{r}=6$

4. Graph each of the solutions in Question 3.

5. Solve the following inequations by using a systematic method. Show your work.

   a. $a+8<2$ b. $r-5>3$
   c. $2b<10$ d. $3m>15$

6. Graph each of the solutions in Question 5.

7. Solve the following problems by developing an equation for each.

   a. Five less than a number is nine.
   b. Three times a number is twenty-four.
   c. Five times a number, increased by four, is nineteen.
   d. Three less than twice a number results in the number plus five.

8. Solve the following problems by using equations.

   a. Lu-Lin spent \$5 less than Ryan. Together they spent \$17.96. How much did each spend?

   b. The sum of two consecutive numbers is -25. What are the numbers?

9. Solve the following problems using a method of your choice.

   a. Jennifer is three times as old as Zoe. Six years ago the sum of their ages was forty-eight. How old are they now?

   b. Jamil had twenty-nine coins in dimes and quarters. He had one more dime than quarter. If the value of the coins is $5, how many coins of each type did Jamil have?

10. Graph the equation $y = 3x - 5$.

11. Solve each of the following equations for y.

   a. $y + x = 2$

   b. $3y = 2x$

12. Graph the equations in Question 11.

13. Harold unpacks cans from boxes at a grocery store.

Number of Cans (c)	Number of Boxes (b)	Relation
12	1	12×1
24	2	12×2
36	3	12×3
48	4	12×4
60	5	12×5

   a. Use words to describe the relationship.
   b. Write an equation to describe the relationship.
   c. Use ordered pairs to describe the relationship.
   d. Use a graph to describe the relationship.

14. Use each of the following tables of values to find each value of $\blacksquare$.

   a.

$a = \blacksquare b + 5$	
a	b
7	1
9	2
11	3
13	4

   b.

$y = 3x - \blacksquare$	
x	y
1	−1
2	2
3	5
4	8

15. A person's maximum heartbeat (b) per minute during exercise can be determined using the following formula where a represents the age of the person in years.

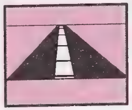
$$b = \frac{4(220 - a)}{5}$$

   a. Find the maximum heartbeat during exercise for a person who is 15 years old.

   b. If the maximum heartbeat during exercise is 144 beats per min, how old is the person?

See your learning facilitator to check your answers and to receive further instructions.

EQUATIONS



What Lies Ahead

In this section you will learn these skills.

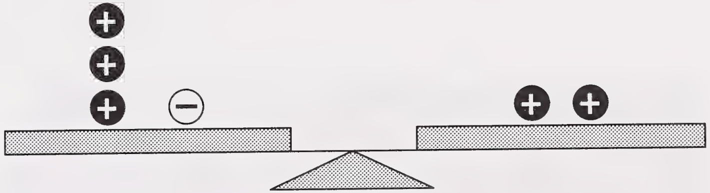
- interpreting equations
- interpreting conditional equations
- modelling equations
- solving equations by inspection or by using the guess-check-revise method
- verifying solutions to equations
- translating English sentences into equations

Working Together

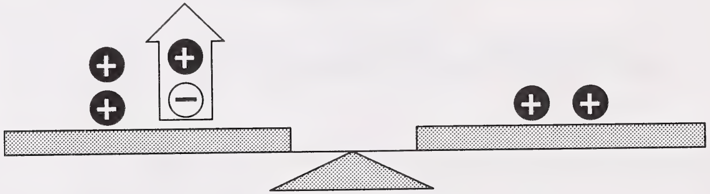
The symbol for equality is =.

An **equation** is any mathematical statement in which two mathematical expressions are connected by the equality symbol. In other words, the value of each side of the equation is the same. For example, $3+(-1)=2$ is an equation.

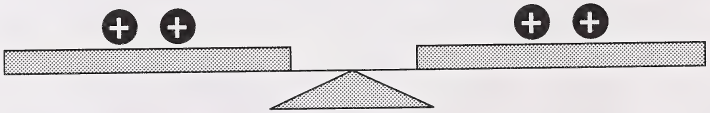
The equation $3+(-1)=2$ can be modelled like this.



The equation can be simplified by removing the zeros (items that combine to equal zero).



The result is this.



Note: The left-hand side (LS) of the scale has a value of $+2$. The right-hand side (RS) of the scale also has a value of $+2$. Therefore, the LS and RS are balanced.

Some equations contain variables. These equations are called **algebraic equations** or **conditional equations**. These algebraic equations are only true for certain values of the variables.

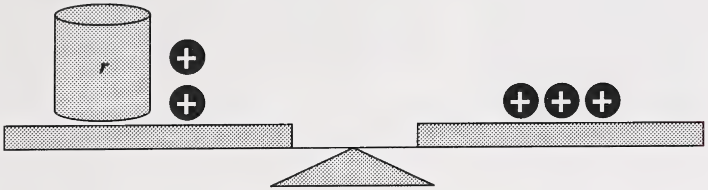
Solving the equation means finding the value of the variable which makes the equation a true statement. This value is called the **solution**.

Example 1

Solve the equation $r + 2 = 3$.

Solution

Model the equation $r + 2 = 3$.

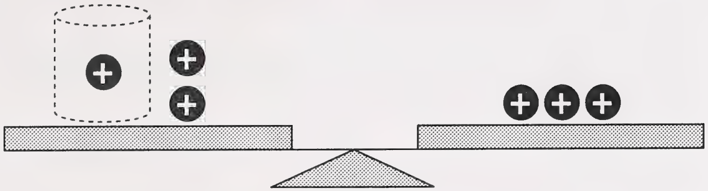


Guess what value of the variable makes the equation a true statement.

$r = 1$

Then verify the solution. **Verifying the solution** means testing to see if the solution results in a true statement.

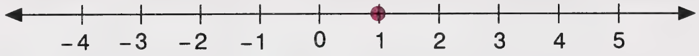
To check, replace the r with $+1$.



Each side of the scale has a value of $+3$ if $r = +1$. The scale is balanced.

So, when $r = +1$, the equation is a true statement.

The solution can be shown by this graph.



Video Activity

Watch the beginning of the video *MATH MOVES: Equations – Solving With One Step* to the end of the segment entitled "Guess and Test". Use the cut-out scale, counter, and cylinders from the Appendix to do the video assignment.

Working Together

Now you will solve equations without using models.

Example 1

Solve the equation $y + 5 = 12$.

Solution

Guess what value of the variable makes the equation true.

5 added to what number gives 12?

$\boxed{7} + 5 = 12$

So, $y = 7$.

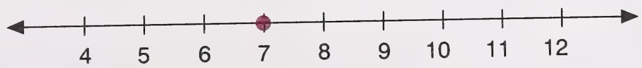
Check

LS	RS
$y+5$	12
$=7+5$	
$=12$	

LS = RS

So, $y=7$.

The solution can be shown by this graph.



Example 2

Solve the equation $5k=55$.

Solution

Guess what value of the variable will make the equation true.

5 times what number is 55?

$5 \times \boxed{11} = 55$

So, $k=11$.

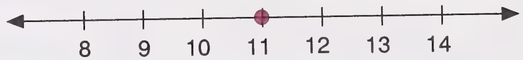
Check

LS	RS
$5k$	55
$=5\times 11$	
$=55$	

LS = RS

So, $k=11$.

The solution can be shown by this graph.



Practice Activity 1

1. Solve the following equations by using inspection or guess-check-revise methods. **Do not** use models.

   a. $n+8=12$ **b.** $p-5=-5$ **c.** $4b=20$

   d. $3b=-12$ **e.** $3w+8=5$ **f.** $4t-1=7$

   g. $5x+4=4x+10$ **h.** $2(a-7)=4$

2. Draw a graph to show each solution in Question 1.

Turn to the Appendix to check your answers.

Working Together

In algebra, English words and phrases can be translated into mathematical expressions with variables. Learning to translate English phrases into mathematical expressions is somewhat like learning a new language.

Translating Words and Phrases

Can you read each of these signs?

Deux et trois

Zwei und drei

Two plus three

Dos y tres

Different languages have been used, but all the signs say the same thing. Each sign can be represented by this mathematical expression.

$2 + 3$

Mathematics is a common language understood by mathematicians all over the world, no matter what language they speak.

You should be aware that different English words or phrases can be translated into the same mathematical expression.

Example

- add two and three
- the sum of two and three
- two plus three
- two increased by three
- two more than three

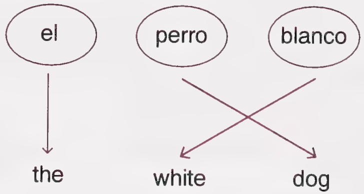
Each of these statements can be translated into this mathematical expression.

$2 + 3$

When you translate from one language to another, you sometimes need to change the order of the words in the new language.

Example

A word-for-word English translation of the Spanish phrase *el perro blanco* would be *the dog white*, but you would say *the white dog* if you were using the rules of English grammar.



When you translate some English phrases into mathematical expressions, you may also need to change the order to reflect the meaning.

Example

- two years | more than | three years

  3 + 2

- one nickel | less than | five nickels

  5 − 1

Note: Because of the commutative property for addition, you can translate *two years more than three years* as $3+2$ or $2+3$. However, the commutative property does not work for subtraction. Therefore, be careful when translating phrases involving subtraction. You must translate *one nickel less than five nickels* as $5-1$.

In spoken English pauses are used to avoid confusion and to group ideas. In written English punctuation marks such as commas are used.

Example

She recognized the boy, who entered the room and gasped.

She recognized the boy who entered the room, and gasped.

Note: In the first sentence the boy gasped. In the second sentence the girl gasped.

In mathematics parentheses are used to show grouping and to indicate the order of operations.

Example 1

The sum of eight, and two times six, is twenty.

$$8 + (2 \times 6) = 20$$

The sum of eight and two, times six, is sixty.

$$(8+2) \times 6 = 60$$

Example 2

The square of nine, minus three, is seventy-eight.

$$9^2 - 3 = 78$$

The square of, nine minus three, is thirty-six.

$$(9-3)^2 = 36$$

Many English phrases can be translated by using variables.

Example

Phrase	Variable
a number	n
the length	ℓ
Adam's mass	a

Phrase	Variable
Bernice's mass	b
Ramon's age	r
Jerry's age	j

Notice that the variable is often the first letter of a word and lower case letters are usually selected.

Get in the habit of writing, rather than printing, variables.

- Do not confuse the number 0 and the variable o.
- Do not confuse the number 1 and the variable ℓ.

Once you have determined the variable, you can write an algebraic equation.

Sentence	Equation
A number increased by six gives thirteen.	$n + 6 = 13$
The length decreased by 4 cm results in 25 cm.	$\ell - 4 = 25$
Five times Adam's mass is 55 kg.	$5a = 55$
Ramon's age divided by four is 7.	$r \div 4 = 7$

Notice that *gives*, *results in*, *is the same as*, and *is* can all be translated into an equal sign (=).

Multiplication

Remember that there are different ways to translate mathematical expressions involving multiplication.

Method 1

When translating expressions such as *five times Adam's mass*, you usually would not use a multiplication sign between the number and the variable.

$$5a$$

If you do include the multiplication sign, be careful that it is not confused with the variable x.

$$5 \times a$$

When translating expressions such as *three times six*, you must use a multiplication sign between the numbers.

$$3 \times 6$$

If you did not include the multiplication sign, it would look like this.

36 ← This is read as "thirty-six", not as "three times six".

Method 2

Brackets can also be used to represent multiplication.

$(3)(6)$ $\qquad$ $(5)(a)$

Method 3

Raised dots are sometimes used to show multiplication.

$3 \cdot 6$ $\qquad$ $5 \cdot a$

If you use the raised dot, be careful that it is not mistaken for a decimal point.

Division

There are different ways to translate mathematical expressions involving division.

Method 1

When translating expressions like *Ramon's age divided by four*, you may use a division sign between the numbers.

$r \div 2$

Method 2

You may also use a fraction to represent division.

$\frac{r}{2}$

Practice Activity 2

1. Write each of the following sentences as an equation.

   a. Twelve decreased by a number is four.

   b. The sum of a number and one-half of the number is equal to forty-eight.

   c. Two less than a number results in seven.

   d. Three kilograms more than Russell's mass is fifty-two kilograms.

   e. Five dollars more than double Mila's money is $79.

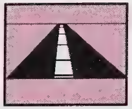
   f. Nine less than half the total number of newspapers delivered is twenty-seven newspapers.

2. Write each of the following sentences as an equation.

   a. Five times a number, increased by eight, is 38.
   b. Five times the sum of a number and eight is 55.
   c. Five times, a number increased by eight, is 45.
   d. Five more than eight times a number is 29.
   e. Eight times the sum of a number and five is 30.
   f. Twice the sum of five times a number and eight is 24.

Turn to the Appendix to check your answers.

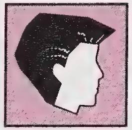
INEQUATIONS



What Lies Ahead

In this section you will learn these skills.

- interpreting inequations
- modelling inequations
- solving inequations by inspection or by using the guess-check-revise method
- verifying solutions to inequations



Working Together

The symbols for inequality are $\neq$, $>$, and $<$.

$\neq$ means "is not equal to".
$>$ means "is greater than".
$<$ means "is less than".

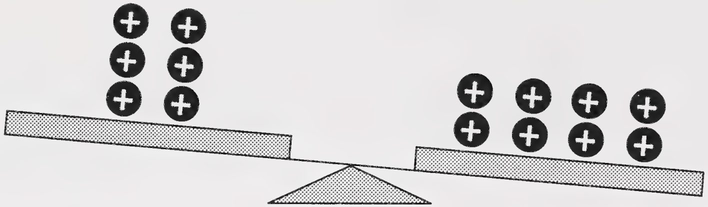
An **inequation** is a mathematical statement in which two expressions are connected by a symbol of inequality.

A scale can be used to model an inequation.

Example 1

$6 < 8$ is an inequation.

It can be modelled like this.

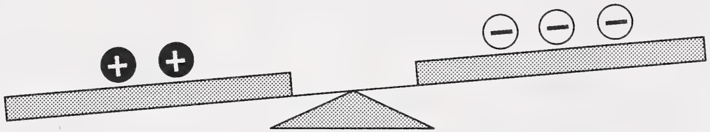


Notice that the value of the left-hand side is less than the value of the right-hand side.

Example 2

$2 > -3$ is an inequation.

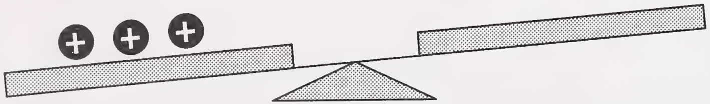
It can be modelled like this.



Notice that the value of the left-hand side is greater than the value of the right-hand side.

Example 3

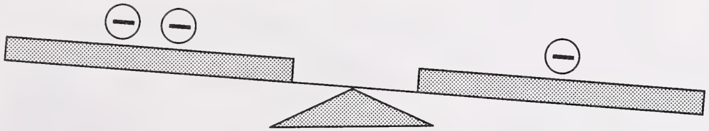
$3 > 0$ is an inequation.



Notice that the value of the left-hand side is greater than the value of the right-hand side.

Example 4

$-2 < -1$ is an inequation.

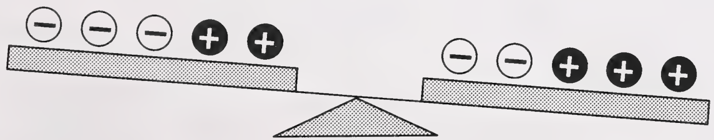


Notice that the value of the left-hand side is less than the value of the right-hand side.

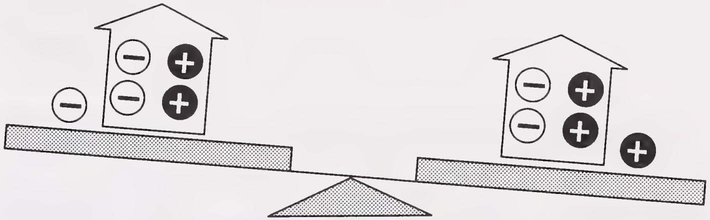
Example 5

$-3 + 2 < -2 + 3$ is an inequation.

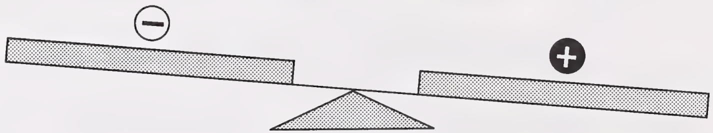
It can be modelled like this.



Simplify the inequation by removing the zeros.



The result is this.



The left-hand side has a value of -1. The right-hand side has a value of $+1$.

The inequation can be simplified as this.

$$-1 < +1$$

Note: The pointed end of the inequality symbol points to the lesser value. The open end of the inequality symbol points to the greater value.

$$-1 < +1$$

Lesser value

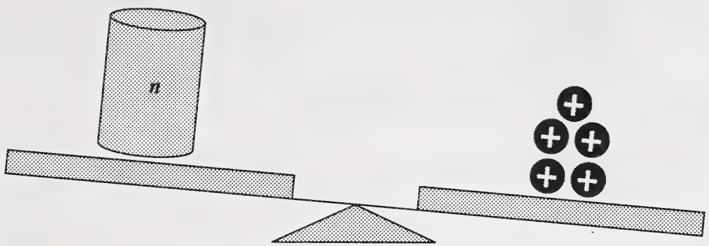
Greater value

Some inequations contain variables.

Example 1: Solve $n < 5$.

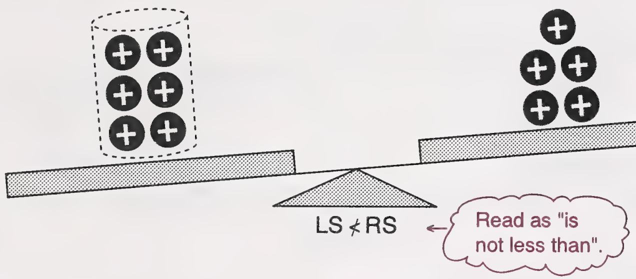
Solution

First model the inequation.



Is $n = 6$?

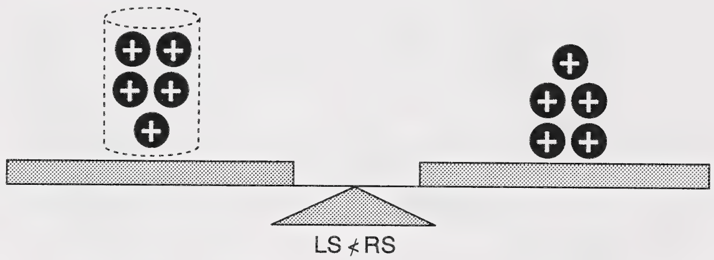
No. You can see that if $n = 6$, the left-hand side will not be less than the right-hand side.



So, $n \neq 6$.

Is $n = 5$?

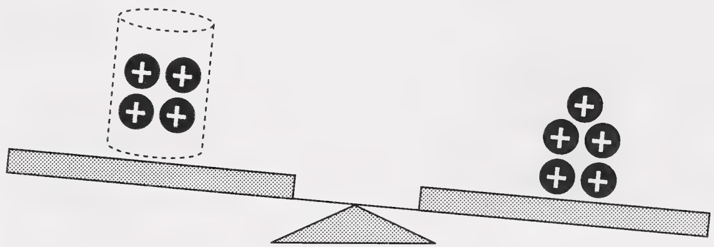
No. You can see that if $n = 5$, the left-hand side will not be less than the right-hand side.



So, $n = 5$ is not a solution.

Is $n = 4$?

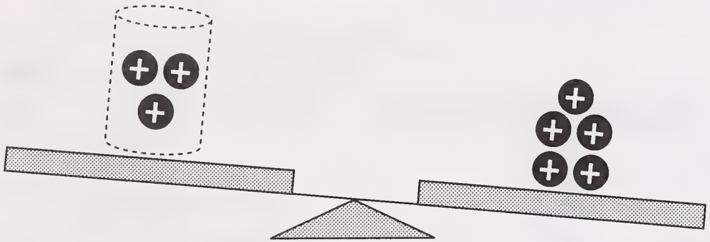
Yes. You can see that if $n = 4$, the left-hand side will be less than the right-hand side.



So, $n = 4$. However, $n = 4$ is not the only solution.

Is $n = 3$?

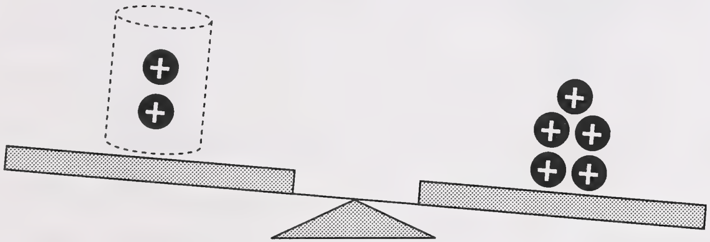
Yes. You can see that if $n = 3$, the left-hand side will be less than the right-hand side.



So, $n = 3$.

Is $n = 2$?

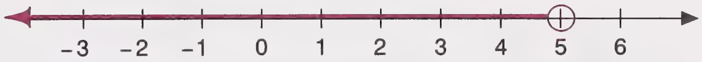
Yes. You can see that if $n = 2$, the left-hand side will be less than the right-hand side.



So, $n = 2$.

In fact, the number of solutions is infinite.

The solutions can be shown by this graph.

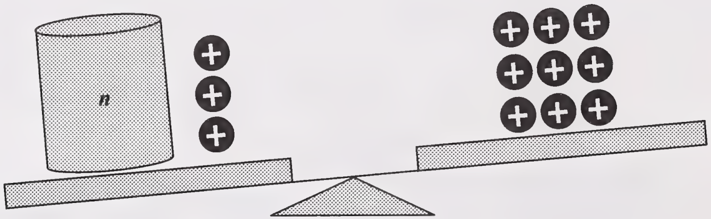


Note: The open circle at 5 indicates that 5 is not a solution. The solid line indicates that any value less than 5 is a solution. In other words, possible solutions include $4\frac{1}{2}$, 2, 0, −1, −2.5, and so on. The list is limitless.

Example 2: Solve $n + 3 > 9$.

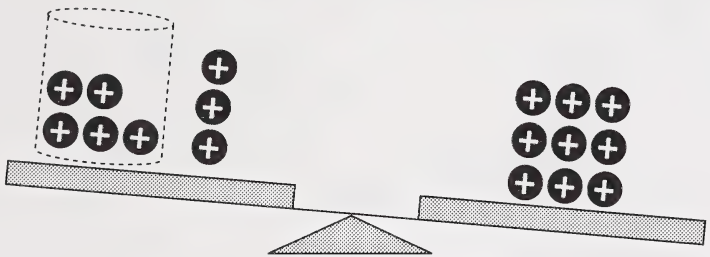
Solution

First model the inequation.



Is $n = 5$?

No. You can see that if $n = 5$, the left-hand side will not be greater than the right-hand side.

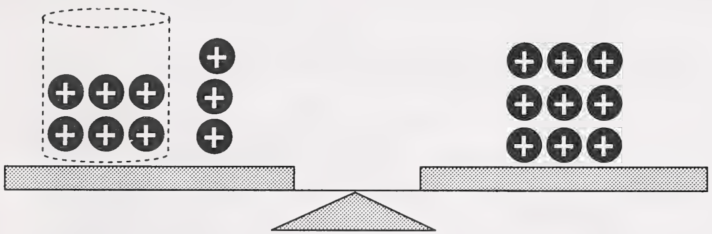


The value of the left-hand side is 8. The value of the right-hand side is 9.

So, $n \neq 5$.

Is $n = 6$?

No. You can see that if $n = 6$, the left-hand side will not be greater than the right-hand side.

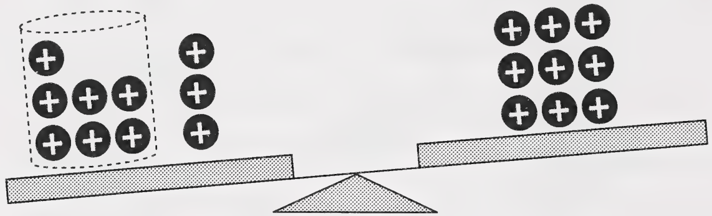


The value of the left-hand side is 9. The value of the right-hand side is 9.

So, $n \neq 6$.

Is $n = 7$?

Yes. You can see that if $n = 7$, the left-hand side will be greater than the right-hand side.

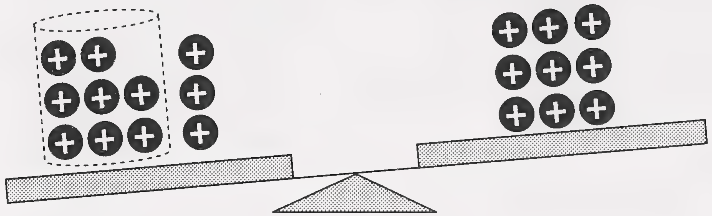


The value of the left-hand side is 10. The value of the right-hand side is 9.

So, when $n = 7$, the statement $n + 3 > 9$ is true.

Other values of n will also make the left-hand side greater than the right-hand side.

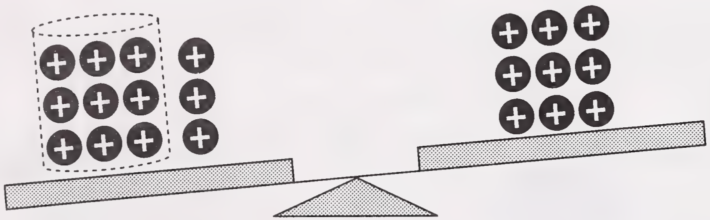
Is $n = 8$?



The value of the left-hand side is 11. The value of the right-hand side is 9.

So, when $n = 8$, the statement $n + 3 > 9$ is true.

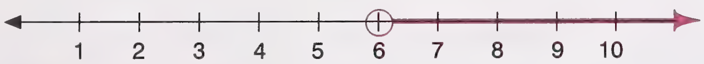
Is $n = 9$?



The value of the left-hand side is 12. The value of the right-hand side is 9.

So, when $n = 9$, the statment $n + 3 > 9$ is true.

The possible values for n can be shown on a graph.

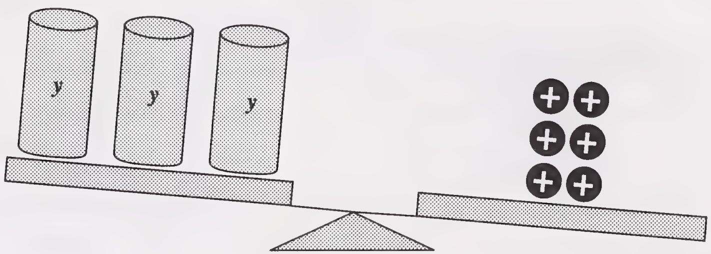


Note: The open circle at 6 indicates that 6 is not part of the solution. The solid line indicates that any value greater than 6 is a solution.

Example 3: Solve $3y < 6$.

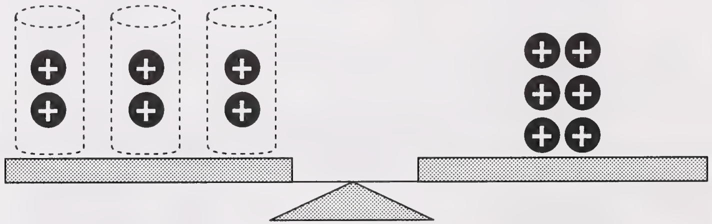
Solution

First model the inequation.



Is $y = 2$?

No. You can see that if $y = 2$, the scale will balance.

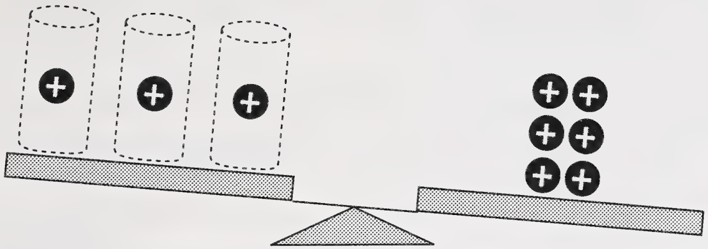


The value of the left-hand side is 6. The value of the right-hand side is 6.

So, $y \neq 2$.

In order to keep the left-hand side less than the right-hand side, y must be less than 2.

Is $y = 1$?

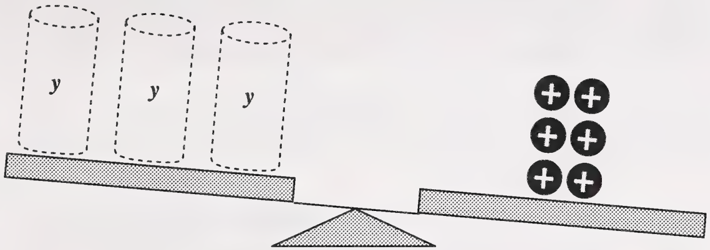


The value of the left-hand side is 3. The value of the right-hand side is 6.

So, when $y = 1$, the statement $3y < 6$ is true.

Other values of y will also make the left-hand side less than the right-hand side.

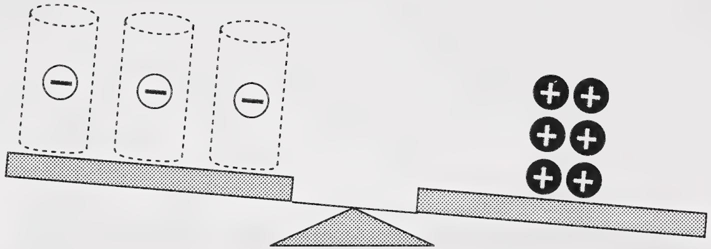
Is $y = 0$?



The value of the left-hand side is 0. The value of the right-hand side is 6.

So, when $y = 0$, the statement $3y < 6$ is true.

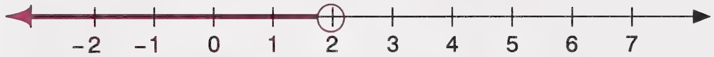
Is $y = -1$?



The value of the left-hand side is -3. The value of the right-hand side is 6.

So, when $y = -3$, the statement $3y < 6$ is true.

The possible values for y can be shown on a graph.



Note: The open circle at that 2 indicates 2 is not part of the solution. The solid line indicates that any value less than 2 is a solution.

Practice Activity 1

Use the cut-out scales, counters, and cylinders from the Appendix for this activity.

1. **a.** Model $m - 5 > 8$.
   b. Solve the inequation by inspection or guess and test.
   c. Draw a graph to show the solutions.

2. **a.** Model $2m < -14$.
   b. Solve the inequation by inspection or guess and test.
   c. Draw a graph to show the solutions.

3. **a.** Model $m + 5 < -2$.
   b. Solve the inequation by inspection or guess and test.
   c. Draw a graph to show the solutions.

4. **a.** Model $3m > -9$.
   b. Solve the inequation by inspection or guess and test.
   c. Draw a graph to show the solutions.

Turn to the Appendix to check your answers.

Working Together

Now you will solve inequations without using models.

Example: Solve $y + 5 < 12$.

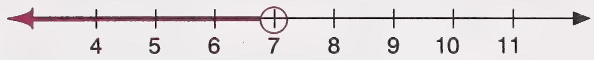
Solution

Guess what values of the variable make the inequation true.

Is $8 + 5 < 12$? No, $13 > 12$. Is $6 + 5 < 12$? Yes, $11 < 12$.
Is $7 + 5 < 12$? No, $12 = 12$. Is $5 + 5 < 12$? Yes, $10 < 12$.

So, when $y < 7$, the statement $y + 5 < 12$ is true.

The solution can be shown by this graph.



Note: The open circle at 7 indicates that 7 is not a solution. The solid line indicates that all the numbers less than 7 are solutions.

Practice Activity 2

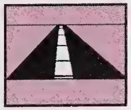
1. Solve each of the following inequations by inspection. **Do not** use models; use only paper-and-pencil methods.

   a. $x + 3 < 6$ **b.** $k - 4 > 3$
   c. $3k < 6$ **d.** $5x > 10$

2. Draw graphs for the solutions in Question 1.

Turn to the Appendix to check your answers.

SOLVING EQUATIONS OF THE FORM $x + a = b$



What Lies Ahead

In this section you will learn these skills.

- using formal procedures to solve equations of the form $x + a = b$
- solving problems using equations

Working Together

In Section 2 you solved equations using inspection and guess-check-revise methods. In this section and later sections you will learn more systematic ways to solve equations.

In Module 2 you learned about **additive inverses**.

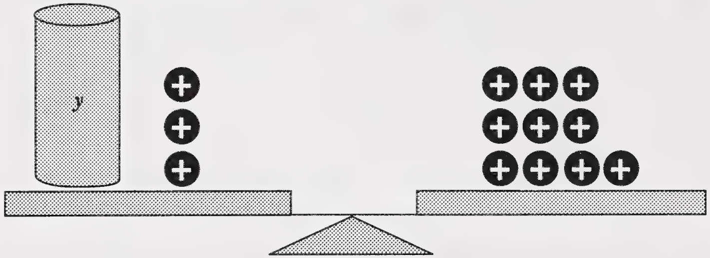
Because their sum is 0, $+3$ and -3 are additive inverses.

You can use additive inverses to solve equations.

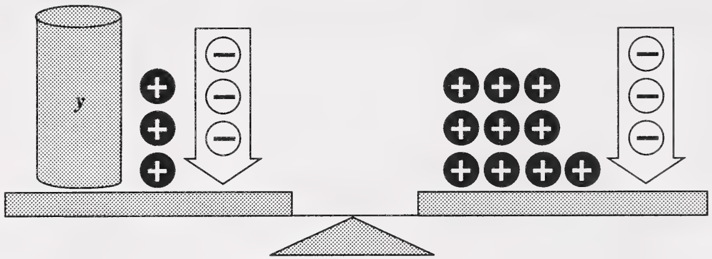
Example 1: Solve the equation $y + 3 = 10$.

Solution

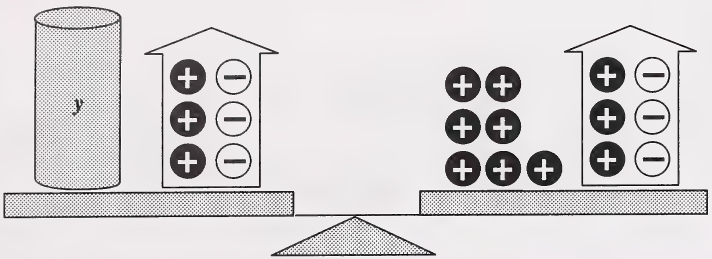
First model the equation.



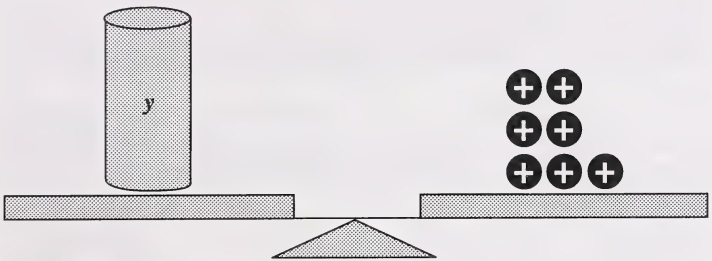
In order to isolate y, add -3 (the additive inverse of $+3$). To maintain equality, add -3 to both sides.



Simplify the equation by removing the zeros. This will not change the balance.

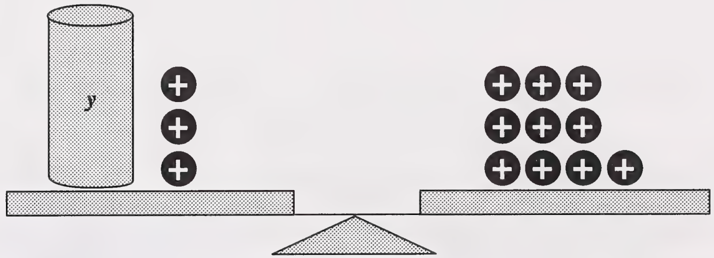


The result is this.

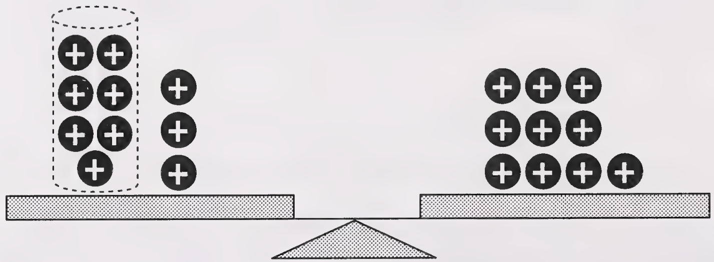


So, $y = 7$.

To verify the solution of $y + 3 = 10$, first model the equation.



Then replace y with $+7$.

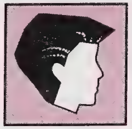


Each side of the scale has a value of $+10$ if $y = +7$. The scale is balanced.

So, $y = 7$.

Video Activity

Watch the third segment of the video *MATH MOVES: Equations – Solving With One Step*. It is entitled "Solving Equations Using Additive Inverses: Equations of Form $x + a = b$". Use the cut-out scales, counters, and cylinders from the Appendix to do the video assignment.



Working Together

Solving Equations Using Paper-and-Pencil Methods

It is not always convenient to rely on learning aids. So, you will now learn a paper-and-pencil method for solving equations.

Example 1: Solve $w - 3 = -2$.

Solution

First write the equation.

$$w - 3 = -2$$

Next isolate the variable by adding $+3$ (the additive inverse of -3) to both sides. You may use this vertical method of adding.

$$\begin{array}{rl} w - 3 = & -2 \\ +3 = & +3 \\ \hline \end{array}$$

Simplify the left-hand side of the equation by removing the zeros. This will not affect the balance.

$$\begin{array}{rl} w \cancel{-3} = & -2 \\ \cancel{+3} = & +3 \\ \hline w \quad = & \end{array}$$

Simplify the right-hand side of the equation to find the solution.

$$\begin{array}{rl} w \cancel{-3} = & -2 \\ \cancel{+3} = & +3 \\ \hline w \quad = & +1 \end{array}$$

So, $w = +1$.

Verification

First write the equation in a chart.

LS	RS
$w - 3$	-2
$= w + (-3)$	

Then replace w with $+1$ and simplify the left-hand side of the equation.

LS	RS
$w - 3$	-2
$= w + (-3)$	
$= 1 + (-3)$	
$= -2$	

LS = RS

So, $w = 1$.

Example 2: Solve the equation $j + 4 = -6$.

Solution

First write the equation.

$$j + 4 = -6$$

Next isolate the variable by adding -4 (the additive inverse of $+4$) to both sides. You may use this horizontal method of adding.

$$j + 4 - 4 = -6 - 4$$

Simplify the left-hand side of the equation by removing the zeros. This will not affect the balance.

$$j \cancel{+4} \cancel{-4} = -6 - 4$$
$$j =$$

Simplify the right-hand side of the equation.

$$j \cancel{+4} \cancel{-4} = -6 - 4$$
$$j = -10$$

Verification

Write the equation in a chart.

LS	RS
$j + 4$	-6

Then replace j with -10 and simplify the left-hand side of the equation.

LS	RS
$j + 4$	-6
$= -10 + 4$	
$= -6$	

LS = RS

So, $j = -10$.

Practice Activity 1

1. What number should be added to both sides to isolate the variable in each of the following equations?

   a. $x + 2 = 7$ **b.** $s + 4 = 9$ **c.** $m + 9 = -13$

   d. $t - 5 = 7$ **e.** $y - 2 = -8$

2. Solve the equations in Question 1 using paper-and-pencil methods. Verify your solutions.

Turn to the Appendix to check your answers.

Working Together

Solving Equations by Working Backwards

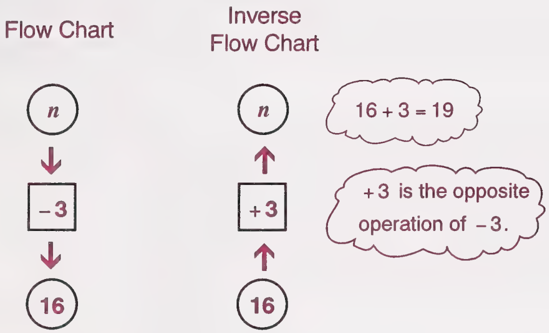
Another method used to solve equations is to work backwards by using flow charts and inverse flow charts.

Remember that the operations of addition and subtraction are opposites, and the operations of multiplication and division are opposites.

Also recall what you learned about flow charts and inverse flow charts in Module 1.

Example 1: Solve $n - 3 = 16$.

First write the equation in flow-chart form. To solve the equation, use an inverse flow chart and work backwards.



So, $n = 19$.

Verify your answer with a chart.

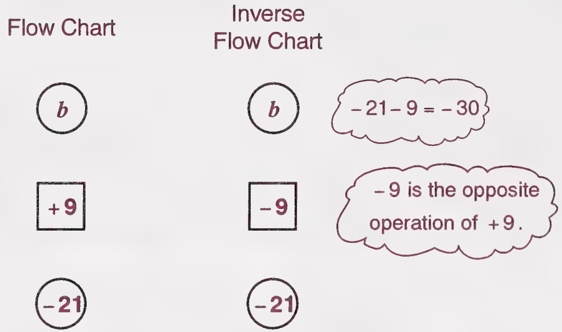
LS	RS
$n - 3$	16
$= 19 - 3$	
$= 16$	

LS = RS

The solution is $n = 19$.

Example 2: Solve $b + 9 = -21$.

First write the equation in a flow chart. To solve the equation, use an inverse flow chart and work backwards.



So, $b = -30$.

Verify your answer with a chart.

LS	RS
$b+9$	-21
$=-30+9$	
$=-21$	

LS = RS

The solution is $b=-30$.

Practice Activity 2

1. Use flow charts and inverse flow charts to solve these equations.

   a. $s-3=5$ **b.** $k+12=39$
   c. $n+2=8$ **d.** $s-4=3$

2. Solve these equations using a paper-and-pencil method. Verify your solutions.

   a. $n-8.5=12.3$ **b.** $q-\frac{1}{2}=\frac{3}{4}$

   c. $p+7.5=8.2$ **d.** $m-5\frac{3}{4}=3\frac{1}{4}$

3. Do the puzzle "What Do You Call It When Police Interrogate a Cow's Husband?"[1] on the following page.

Turn to the Appendix to check your answers.

Did You Know?

The equal sign was used for the first time in the book *The Whetstone of Wilte* published in 1557. The author, Robert Recorde (1510-1558), an English mathematician, explained that he chose a pair of parallel line segments having the same length to represent the equality sign because he believed that no two things could be more equal.

[1] 1978 Creative Publications for excerpt from *Pre-Algebra with Pizzazz.*

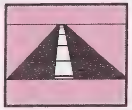
WHAT DO YOU CALL IT WHEN POLICE INTERROGATE A COW'S HUSBAND?

Solve each problem and find the solution in the rectangle below. Cross out the box containing that solution. When you finish, there will be six boxes not crossed out. Print the letters from these boxes in the spaces at the bottom of the page.

1. Eight more than a number is 20. Find the number.
2. Twelve less than a number is -3. Find the number.
3. Three more than a number is -5. Find the number.
4. Nine less than a number is -24. Find the number.
5. If 10 is subtracted from a number, the result is 23. Find the number.
6. If 32 is added to a number, the result is -4. Find the number.
7. If a number is increased by 6, the result is 50. Find the number.
8. If a number is decreased by 16, the result is -2. Find the number.
9. The length of a rectangular lot is 78 m. This is 51 m more than the width. What is the width?
10. Andy hit 14 home runs this season. If this is 9 fewer than he hit last season, how many home runs did he hit last season?
11. Jennifer added $120 to her savings account during July. If this brought her balance to $700, how much had she saved previously?
12. The temperature in Frostburg is -7°C. This is 18°C less than the temperature in Coldspot. Find the temperature in Coldspot.
13. After 9 new members joined the ski club, there were 38 members. How many members had been in the club previously?
14. The altitude of a submarine is -60 m. If this is 25 m less than its previous altitude, what was its previous altitude?

CO 14	IN 33	JA −35 m	WS $580	LK 12	QU −75 m	IT 29	SH −15	AM 9	ES $565
OO −36	TI 8°C	ME 27 m	ON 31	ST −8	TO 11°C	AB 17	OP 44	ED 23	LE 32 m

SOLVING INEQUATIONS OF THE FORM $x+a<b$ OR $x+a>b$



What Lies Ahead

In this section you will learn this skill.

- using formal procedures to solve equations of the form $x + a < b$ or $x + a > b$

Working Together

In the previous section you learned that you can add the same number to both sides of an equation and still keep the equality. You can also subtract the same number from both sides of an equation without changing the equality.

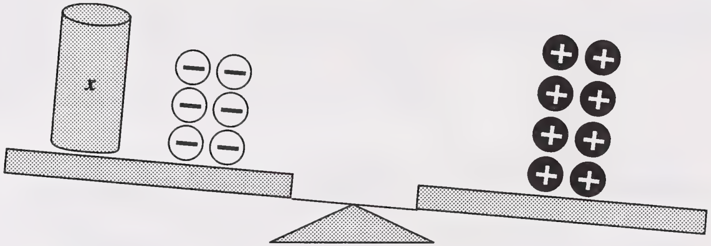
The same property can be applied to inequations. Adding the same number to both sides of an inequation will not change the inequality. Subtracting the same number from both sides of an inequation also keeps the inequality.

Because of this property, you can solve some inequations by using additive inverses.

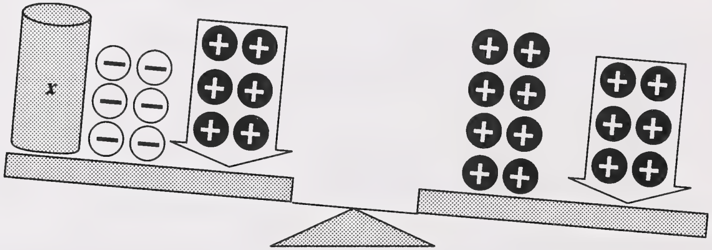
Example 1: Solve $x - 6 < 8$.

Solution

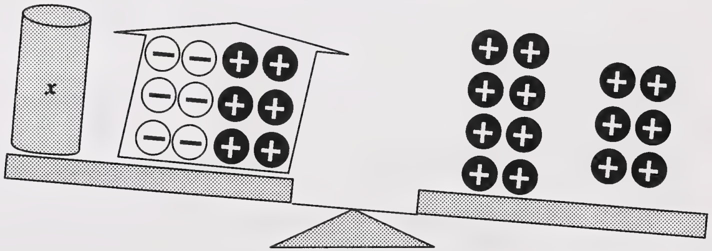
Model the inequation.



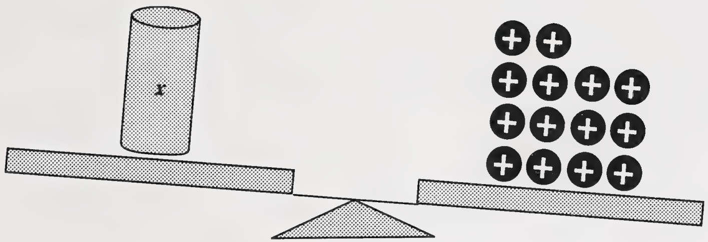
Add $+6$ (the additive inverse of -6) to each side.



Simplify the inequation by removing the zeros.

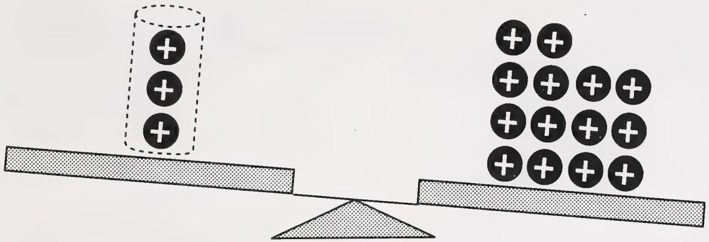


The result is this.



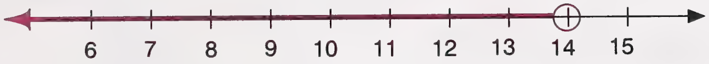
So, $x < 14$.

Verify the solutions. Replace x with any value less than 14, such as 3.



So, when $x < 14$, the statement $x - 6 < 8$ is true.

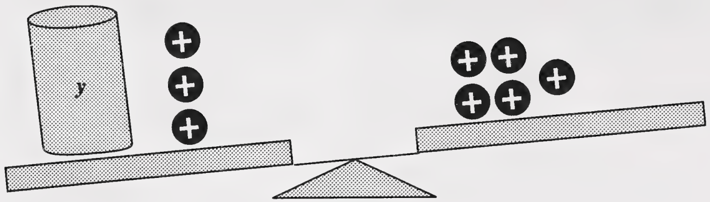
The solutions can be shown in this graph.



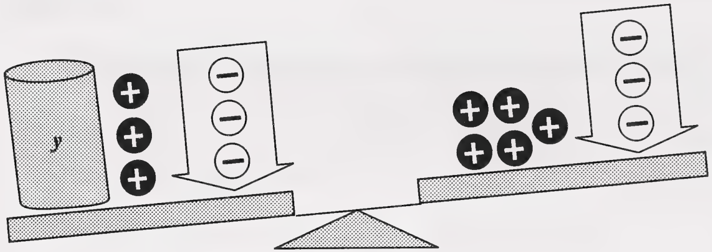
Example 2: Solve $y + 3 > 5$.

Solution

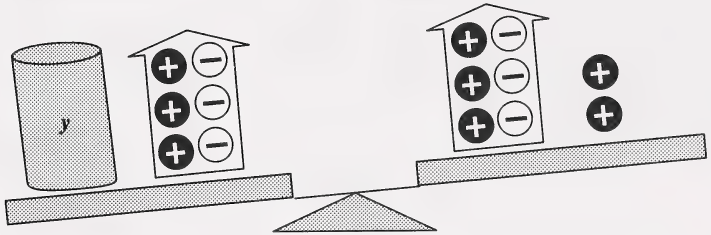
Model the inequation.



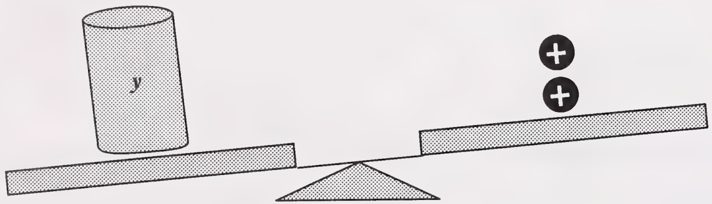
Add -3 (the additive inverse of $+3$) to each side.



Simplify the inequation by removing the zeros.

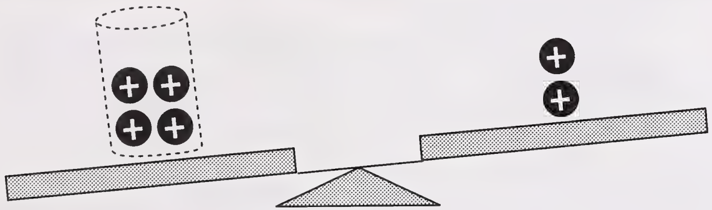


The result is this.



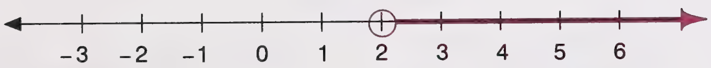
So, $y > 2$.

Verify the solutions with any value greater than 2, such as 4.



So, when $y > 2$, the statement $y + 3 > 5$ is true.

The solutions can be shown in this graph.



Practice Activity 1

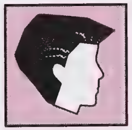
1. Use the cut-out scales, cylinders, and counters from the Appendix to model and solve the following inequations.

   a. $y + 5 > 9$ b. $m - 4 < 6$

   c. $r + 3 < 5$ d. $t - 2 > 8$

2. Draw graphs to show the solutions in Question 1.

Turn to the Appendix to check your answers.



Working Together

It is not always convenient to rely on learning aids. So, now you will use a paper-and-pencil method to solve inequations.

Example: Solve $k - 1 < 12$.

Solution

Isolate the variable by adding 1 (the additive inverse of -1) to each side of the inequation.

$$\begin{array}{l} k \cancel{-1} < 12 \\ \underline{\;\;\cancel{+1}\;\; +1} \\ k \quad\;\; < 13 \end{array}$$

Verify the solution with any value less than 13, such as 12.

LS	RS
$k - 1$	12
$= 12 - 1$	
$= 11$	

LS < RS

So, the solution is $k < 13$.

Practice Activity 2

1. Use paper-and-pencil methods to solve the following inequations. Be sure to verify the solutions.

   a. $m + 7 < 10$ b. $m - 2 > 11$ c. $x + 4 < 6$
   d. $y - 3 > 8$ e. $w + 2 > 6$

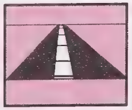
2. Graph the solutions in Question 1.

Turn to the Appendix to check your answers.

Did You Know?

Mathematical symbols have been used for thousands of years. However, the symbols < and > are only about 400 years old. Thomas Harriot, an Englishman, was the first person to use > to mean "is greater than" and < to mean "is less than".

SOLVING EQUATIONS OF THE FORM $ax = b$



What Lies Ahead

In this section you will learn these skills.

- using formal procedures to solve equations of the form $ax = b$
- solving problems using equations

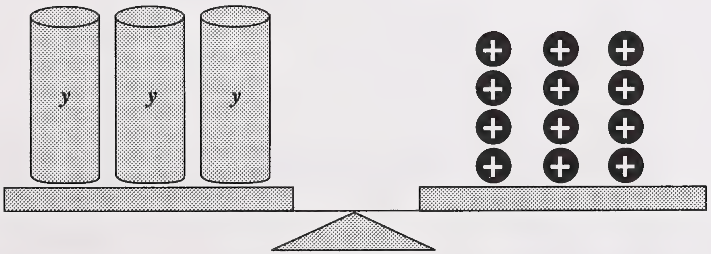
Working Together

You have learned to solve equations using additive inverses. In this section you will learn a method for solving equations using **multiplicative inverses**.

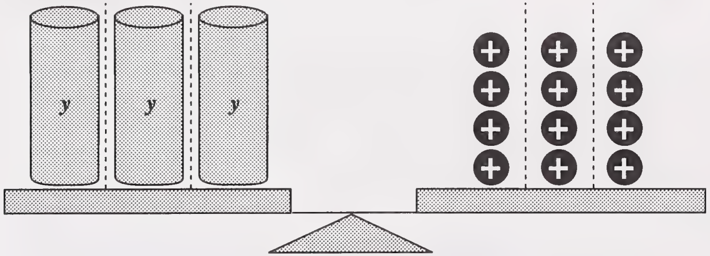
Example 1: Solve the equation $3y = 12$.

Solution

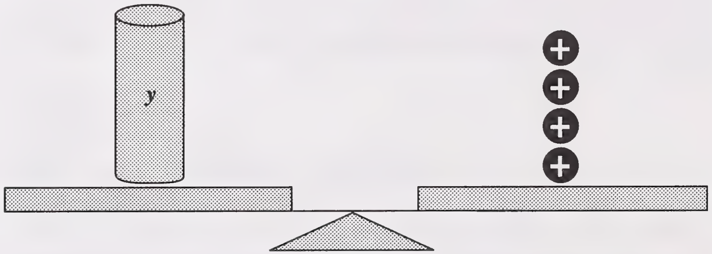
First model the equation.



To isolate the variable, multiply each side by $\frac{1}{3}$ (the multiplicative inverse of 3). Begin by dividing each side into three groups.

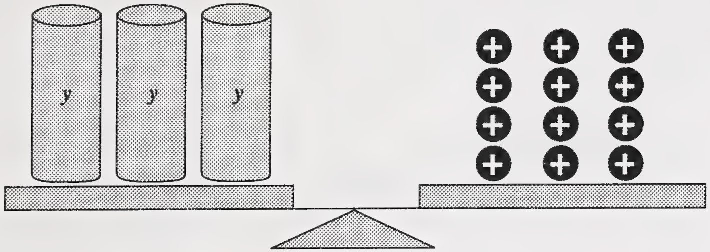


To solve the equation, examine only one of the three groups on each side.

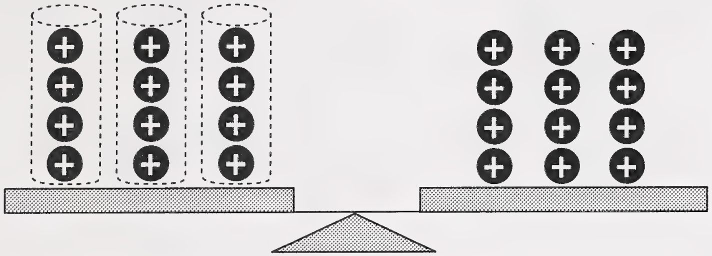


The solution is $y = 4$.

To verify the solution of $3y = 12$, you must first model the equation.



Then replace y with 4.



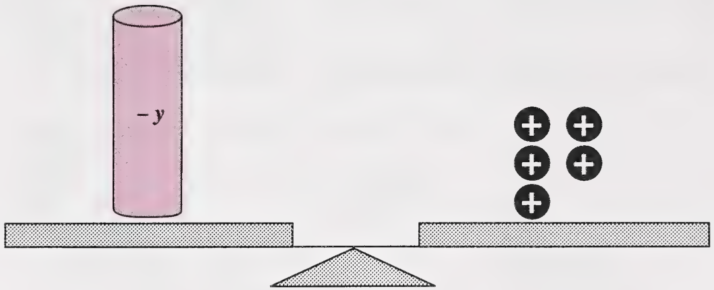
Each side of the equation has a value of $+12$ if $y = 4$. The scale is balanced.

So, $y = 4$.

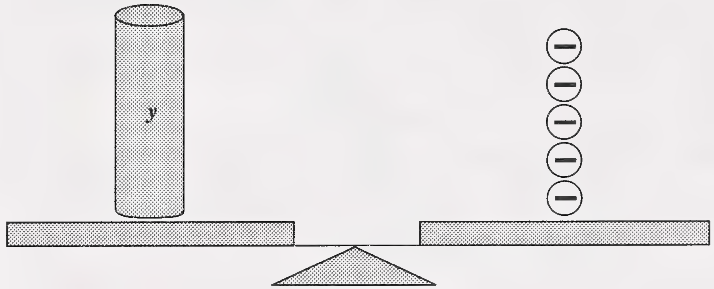
Example 2: Solve the equation $-y = 5$.

Solution

Model the equation.

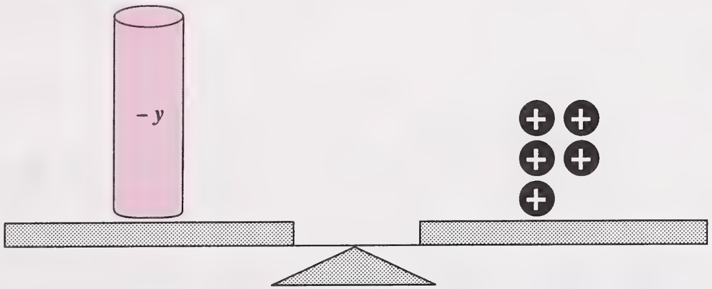


Replace each side with its inverse.

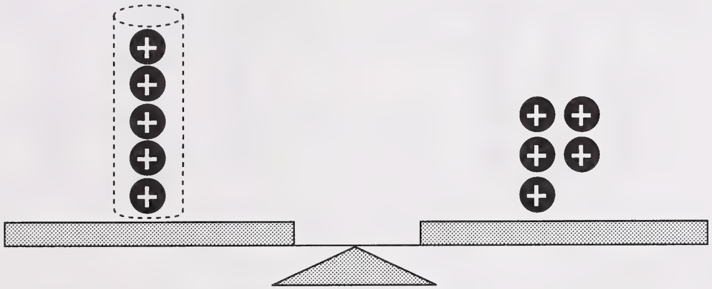


So, $y = -5$.

To verify the solution, you must first model the equation.



If $y = -5$, $-y = 5$. So, replace $-y$ with 5.



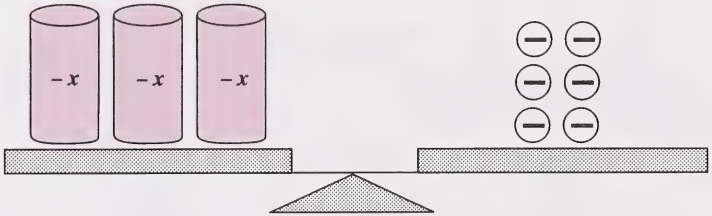
Each side of the equation has a value of $+5$ if $y = -5$. The scale is balanced.

So, $y = -5$.

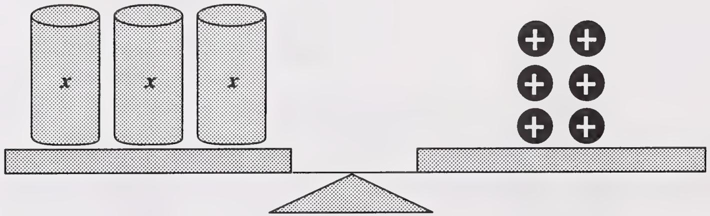
Example 3: Solve the equation $-3x = -6$.

Solution

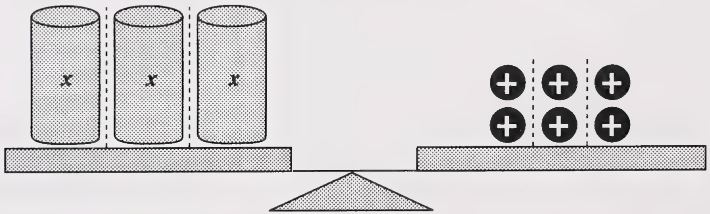
Model the equation.



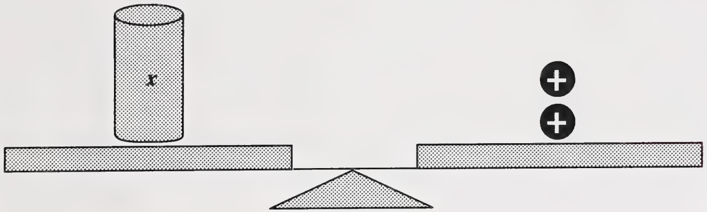
Replace each side with its inverse.



To isolate x, divide each side into three groups.

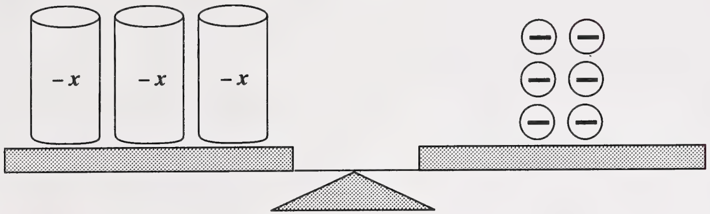


To solve for x, examine one group on each side.

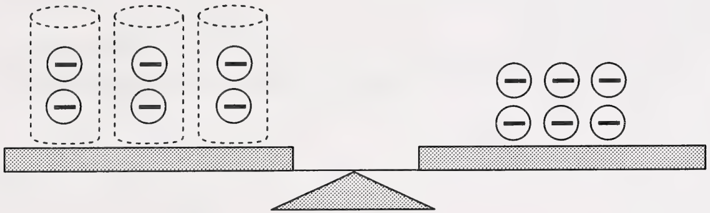


The solution is $x = 2$.

To verify the solution, model the equation.



Since $x = 2$, $-x = -2$. Replace $-x$ with -2.

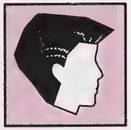


Each side of the equation equals -6. The scale is balanced.

So, the solution is $x = 2$.

Video Activity

Watch the fourth and fifth segments of the video *MATH MOVES: Equations – Solving With One Step*. They are entitled "Solving Equations Using Multiplicative Inverses: Equations of Form $ax = b$" and "Solving Equations With Multiplicative Inverses of -1". Use the cut-out scale, counters, and cylinders from the Appendix to do the video assignment.



Working Together

Solving Equations Using Paper-and-Pencil Methods

It is not always convenient to rely on learning aids, so you will now learn a paper-and-pencil method to solve equations.

Example 1: Solve the equation $4d = -16$.

Solution

First write the equation.

$$4d = -16$$

To isolate the variable, multiply both sides by $\frac{1}{4}$ (the multiplicative inverse of 4).

$$\frac{1}{4} \times 4d = \left(\frac{1}{4}\right)(-16)$$

Simplify each side of the equation.

$$\frac{1}{\cancel{4}_1} \times \cancel{4}^1 d = -\left(\frac{1}{\cancel{4}_1} \times \cancel{16}^4\right)$$

$$1d = -4$$

$$d = -4$$

The solution is $d = -4$.

Verification

First write the equation in a chart.

LS	RS
$4d$	-16

Then replace d with -4 and simplify.

LS	RS
$4d$	-16
$= 4 \times (-4)$	
$= -16$	

LS = RS

So, the solution is $d = -4$.

Example 2: Solve the equation $-3r = -6$

Solution

First write the equation.

$$-3r = -6$$

Replace each side with its inverse.

$$3r = 6$$

Multiply each side by $\frac{1}{3}$ (the multiplicative inverse of 3).

$$\frac{1}{3} \times 3r = \frac{1}{3} \times 6$$

Simplify each side.

$$\frac{1}{\cancel{3}_1} \times \cancel{3}^1 r = \frac{1}{\cancel{3}_1} \times \cancel{6}^2$$

$$r = 2$$

The solution is $r = 2$.

Verification

First write the equation in a chart.

LS	RS
$-3r$	-6

Replace r with 2 and simplify.

LS	RS
$-3r$	-6
$= -3(2)$	
$= -6$	

LS = RS

So, the solution is $r = 2$.

Practice Activity 1

1. What number should both sides be multiplied by to isolate the variable in each of the following equations?

   a. $2y = 18$ b. $4v = 32$

   c. $3m = -9$ d. $2f = -4$

2. Solve the equations in Question 1 by using paper-and-pencil methods. Verify your solutions.

3. Solve the following equations.

   a. $-3a = 18$ b. $-5y = -10$

Turn to the Appendix to check your answers.

Working Together

Solving Equations by Working Backwards

You can solve equations like those in Practice Activity 1 by using flow charts and inverse flow charts.

Example: Solve the equation $4d = 144$.

Solution

First write the equation in a flow chart. To solve the equation, use an inverse flow chart.

Flow Chart	Inverse Flow Chart
d	d
$\downarrow$	$\uparrow$
$\boxed{\times 4}$	$\boxed{\div 4}$
$\downarrow$	$\uparrow$
144	144

$144 \div 4 = 36$

$\div 4$ is the inverse operation of $\times 4$.

So, $d = 36$.

Use a chart to verify your answer.

LS	RS
$4d$	144
$= 4 \times 36$	
$= 144$	

LS = RS

So, the solution is $d = 36$.

Practice Activity 2

1. Use flow charts and inverse flow charts to solve these equations.

   a. $9t = -72$ **b.** $3t = 30$
   c. $-4n = 44$ **d.** $-2w = -10$

2. Solve the following equations using either paper-and-pencil method. Verify the solutions.

   a. $4t = 6$ **b.** $3a = \frac{1}{2}$ **c.** $2r = \frac{3}{4}$
   d. $2p = 14.4$ **e.** $3m = 0.9$

3. Solve the following problems.

   a. Five times a number is 75. Find the number.
   b. Three times a number is -12. Find the number.
   c. Twice a number is -56. Find the number.
   d. Nine times a number is 99. Find the number.

Turn to the Appendix to check your answers.

SOLVING INEQUATIONS OF THE FORM $ax < b$ OR $ax > b$

What Lies Ahead

In this section you will learn this skill.

- using formal procedures to solve inequations of the form $ax < b$ or $ax > b$

Working Together

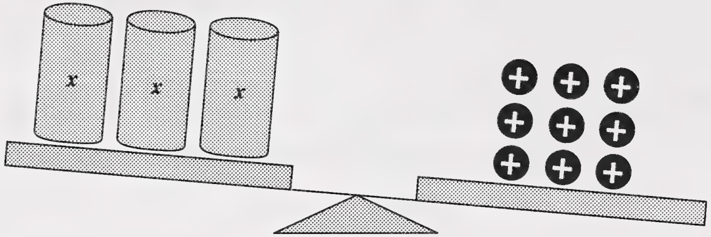
In the previous section you learned that you can multiply both sides of an equation by the same number and still keep the equality. You can also divide both sides of an equation by the same number without changing the equality.

Multiplying or dividing both sides of an inequation by the same *positive* number will not change the inequality. Because of this property, you can solve some inequations by using multiplicative inverses.

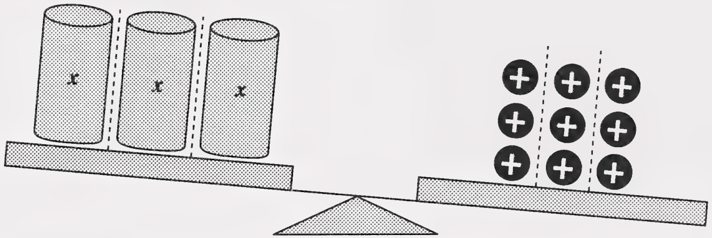
Example: Solve $3x < 9$.

Solution

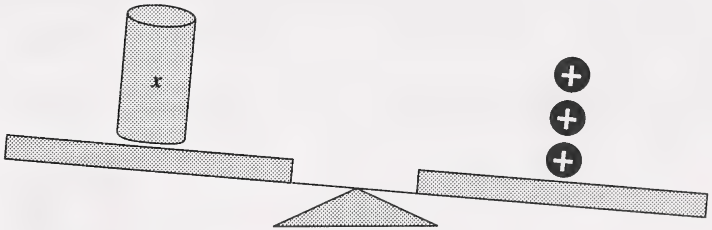
Model the inequation.



Divide each side into three groups.

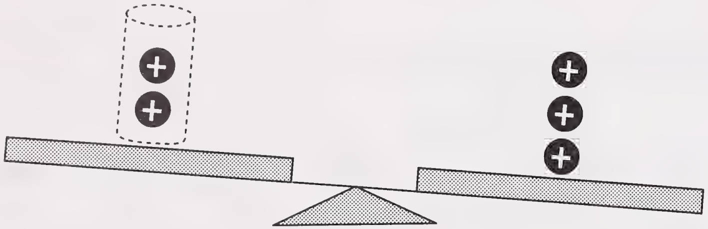


To solve the inequation, examine one group from each side.



So, $x < 3$.

Verify the solution. Replace x with any value less than 3, such as 2.



LS < RS

So, $x < 3$ is the solution.

Practice Activity 1

1. Use the cut-out scales, cylinders, and counters from the Appendix to model and solve the following inequations.

   a. $2m < -8$ **b.** $3m > 12$

   c. $2m < 10$ **d.** $5m > 10$

2. Draw graphs of the solutions in Question 1.

Turn to the Appendix to check your answers.

Working Together

Now you will use paper-and-pencil methods to solve inequations.

Example: Solve $3m < -9$.

Solution

First write the inequation.

$$3m < -9$$

Multiply each side by $\frac{1}{3}$ (the multiplicative inverse of 3).

$$\begin{aligned} 3m &< -9 \\ \frac{1}{3} \times 3m &< \frac{1}{3} \times (-9) \\ m &< -3 \end{aligned}$$

Verify the solution with any value less than -3, such as -4.

LS	RS
$3m$	-9
$= 3 \times (-4)$	
$= -12$	

LS < RS

So, the solution is $m < -3$.

Practice Activity 2

1. Use paper-and-pencil methods to solve the following inequations. Be sure to verify the solutions.

   a. $4m < 12$ **b.** $3m < 27$
   c. $2m > 18$ **d.** $5m > 25$

2. Graph the solutions in Question 1.

Turn to the Appendix to check your answers.

Working Together

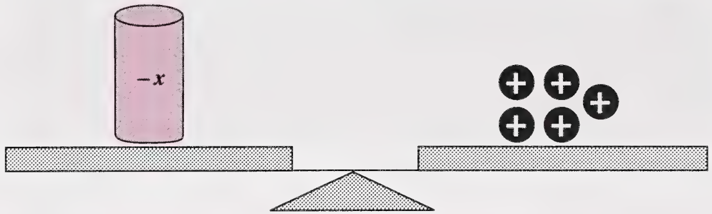
The remainder of the section is included for enrichment.

In the previous section you solved equations like $-x = 5$ and $-2x = 8$.

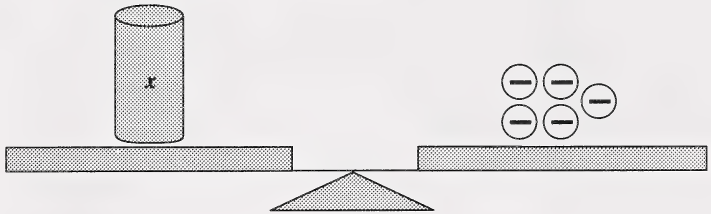
You learned that multiplying each side of an equation by -1 does not change the equality.

Example

$-x = 5$ can be modelled like this.



To solve $-x = 5$, you can multiply each side by -1 (or replace each side with its inverse).

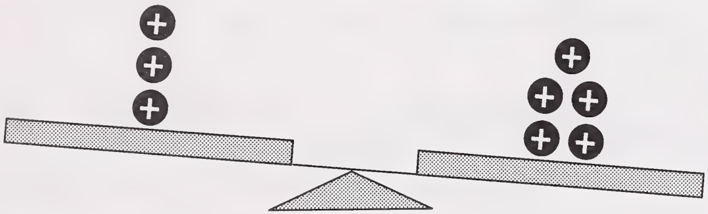


So, $x = -5$.

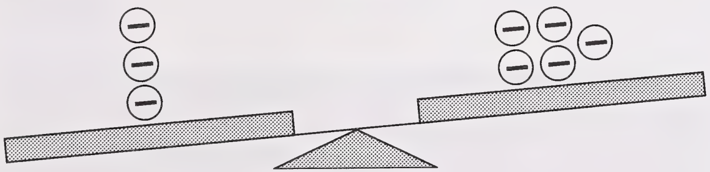
Notice that if you multiply each side of an inequation by -1, the inequality is **not** maintained.

Example 1

$3 < 5$ is an inequation. It can be modelled like this.



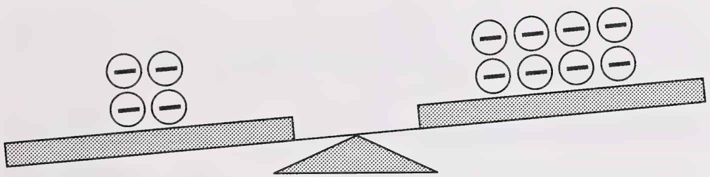
If you multiply each side by -1, or replace each side with its inverse, the inequality changes.



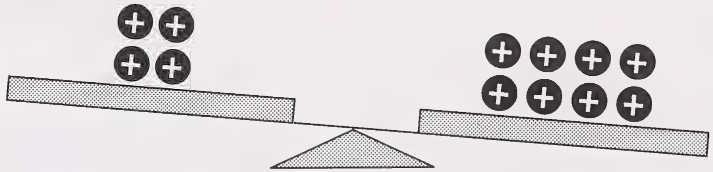
$-3 > -5$

Example 2

$-4 > -8$ is an inequation. It can be modelled like this.



If you multiply each side by -1, or replace each side with its inverse, the inequality changes.



$4 < 8$

Practice Activity 3

Use the cut-out counters and scales from the Appendix to model the following inequations and see what happens to the inequality when you multiply both sides of an inequation by -1.

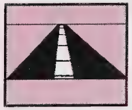
1. $-3 < 2$

2. $5 > 1$

3. $-1 > -4$

Turn to the Appendix to check your answers.

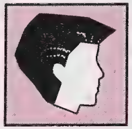
SOLVING EQUATIONS OF THE FORM $\frac{x}{a}=b$, $\frac{x}{a}=\frac{b}{c}$, OR $\frac{a}{x}=\frac{b}{c}$



What Lies Ahead

In this section you will learn this skill.

- using formal procedures to solve equations of the form $\frac{x}{a}=b$, $\frac{x}{a}=\frac{b}{c}$, or $\frac{a}{x}=\frac{b}{c}$



Working Together

In Section 6 you solved equations in which the variable was multiplied by a whole number. Here is an example.

$3a = 6$ — $3a$ means $3 \times a$.

In this section you will learn to solve equations in which the variable is multiplied by a fraction. Here are some examples.

$\frac{a}{2} = 3$ — $\frac{a}{2}$ means $\frac{1}{2}a$ or $\frac{1}{2} \times a$.

$\frac{x}{3} = \frac{1}{2}$ — $\frac{x}{3}$ means $\frac{1}{3}x$ or $\frac{1}{3} \times x$.

You can solve some equations with fractions using multiplicative inverses.

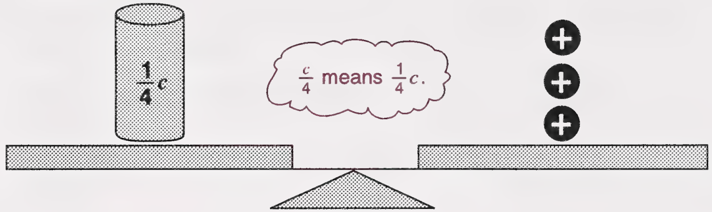
Both the modelling method and the paper-and-pencil method are shown.

Example: Modelling Method

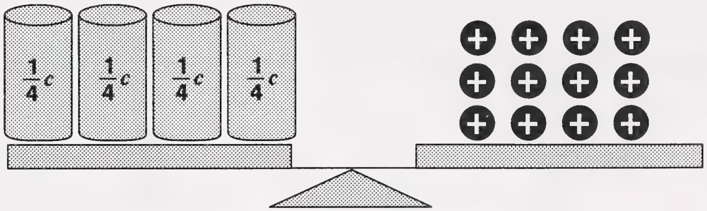
Solve $\frac{c}{4} = 3$ by isolating the variable.

Solution

First model the equation.

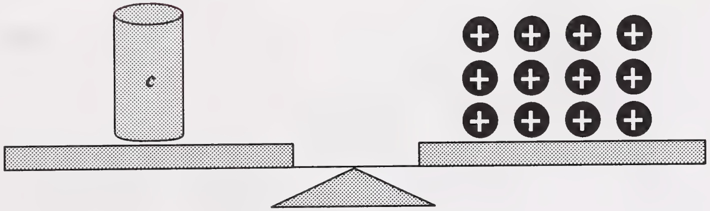


The multiplicative inverse of $\frac{1}{4}$ is 4. So, to isolate c, multiply each side by 4.



Simplify the equation by combining like terms.

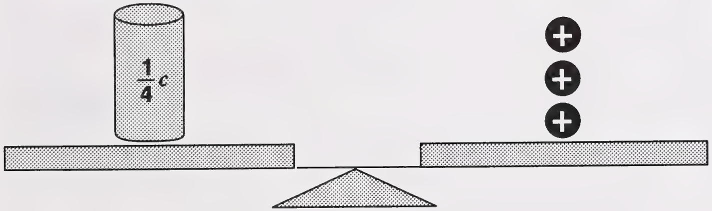
$\frac{1}{4}c + \frac{1}{4}c + \frac{1}{4}c + \frac{1}{4}c = c$



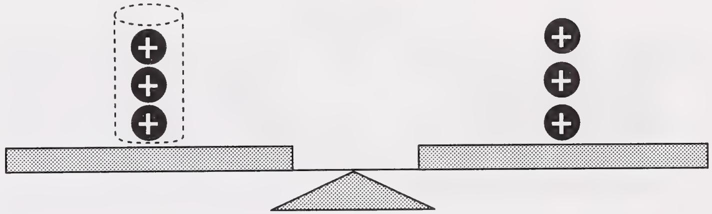
The solution is $c = 12$.

Verification

First model the equation.



Because $c = 12$, replace $\frac{1}{4}c$ with $\frac{1}{4} \times 12$, or 3.



Each side of the equation has a value of $+3$ if $c = 12$. The scale is balanced.

So, $c = 12$.

Example: Paper-and-Pencil Method

Solve $\frac{c}{4} = 3$ by isolating the variable.

Solution

First write the equation.

$$\frac{c}{4} = 3$$

$\frac{c}{4}$ means $\frac{1}{4}c$.

The multiplicative inverse of $\frac{1}{4}$ is 4. So, to isolate the variable, multiply each side by 4.

$$\frac{c}{4} \times 4 = 3 \times 4$$

Simplify the equation by performing the multiplication.

$$\frac{c}{\cancel{4}_1} \times \cancel{4}^1 = 3 \times 4$$

$$c = 12$$

The solution is $c = 12$.

Verification

LS	RS
$\frac{c}{4}$	3
$= \frac{12}{4}$	
$= 3$	

LS = RS

So, the solution is $c = 12$.

Practice Activity 1

1. What number should both sides of each equation be multiplied by to isolate the variable?

   a. $\frac{n}{5} = 6$ b. $\frac{a}{11} = 3$ c. $\frac{r}{2} = -12$

   d. $\frac{d}{8} = 2$ e. $\frac{b}{7} = -1$

2. Solve each of the equations in Question 1 using paper-and-pencil methods.

Turn to the Appendix to check your answers.

Working Together

In the Practice Activity 1 you solved equations that had a fraction on one side of the equation.

Now you will solve equations with fractions on both sides of the equation.

Example

Solve $\frac{p}{5} = \frac{1}{2}$ by isolating the variable.

Solution

Write the equation.

$$\frac{p}{5} = \frac{1}{2}$$

$\frac{p}{5}$ means $\frac{1}{5}p$.

The multiplicative inverse of $\frac{1}{5}$ is 5. So, to isolate the variable, multiply each side by 5.

$$\frac{p}{5} \times 5 = \frac{1}{2} \times 5$$

Simplify the equation by performing the multiplication.

$$\frac{p}{\cancel{5}_1} \times \cancel{5}^1 = \frac{1}{2} \times 5$$

$$p = \frac{5}{2} \text{ or } 2.5$$

Verification

LS	RS
$\frac{p}{5}$	$\frac{1}{2}$
$= \frac{2.5}{5}$	$= 0.5$
$= 0.5$	

LS = RS

So, the solution is $p = 2.5$.

Practice Activity 2

1. What number should both sides of each of the following equations be multiplied by to isolate the variable?

   a. $\frac{n}{15} = \frac{2}{3}$ **b.** $\frac{f}{9} = \frac{8}{3}$ **c.** $\frac{y}{5} = \frac{5}{6}$

   d. $\frac{r}{7} = \frac{5}{8}$ **e.** $\frac{3}{4} = \frac{c}{20}$

2. Solve the equations in Question 1.

Turn to the Appendix to check your answers.

Working Together

Solving Equations by Working Backwards

Flow charts and inverse flow charts are helpful in solving equations.

Remember that $\frac{1}{5}n = \frac{n}{5}$.

Also remember that $\frac{n}{5} = n \div 5$.

Study the following examples.

Example 1: Solve $\frac{x}{5} = 5$.

Solution

First write the equation in a flow chart. To solve the equation, use an inverse flow chart.

Flow Chart	Inverse Flow Chart
x	x
$\downarrow$	$\uparrow$
$\boxed{\div 5}$	$\boxed{\times 5}$
$\downarrow$	$\uparrow$
5	5

$5 \times 5 = 25$

$\times 5$ is the inverse operation of $\div 5$.

So, $x = 25$.

Verification

LS	RS
$\frac{x}{5}$	5
$= \frac{25}{5}$	
$= 5$	

LS = RS

So, the solution is $x = 25$

Practice Activity 3

1. Use flow charts and inverse flow charts to solve these equations.

   a. $\frac{a}{3} = 2$ **b.** $\frac{b}{4} = 16$ **c.** $\frac{c}{2} = \frac{3}{8}$ **d.** $\frac{d}{5} = \frac{8}{25}$

2. Solve the following problems.

   a. One-eighth of a number is five. Find the number.
   b. One-fifth of a number is -2. Find the number.
   c. One-fourth of a number is $\frac{1}{2}$. Find the number.
   d. One-half of a number is $\frac{3}{4}$. Find the number.

Turn to the Appendix to check your answers.

Working Together

Now you will learn two short-cut methods to solve equations of the form $\frac{x}{a} = \frac{b}{c}$ or $\frac{a}{x} = \frac{b}{c}$.

Example 1: Solve $\frac{p}{5} = \frac{1}{2}$.

Solution

Method 1

Write the equation.

$$\frac{p}{5} = \frac{1}{2}$$

Then multiply both sides of the equation by 10 (the product of the denominators 2 and 5). This is called **clearing the denominators**.

$$10 \times \frac{p}{5} = \frac{1}{2} \times 10$$

Simplify each side of the equation.

$$\overset{2}{\cancel{10}} \times \frac{p}{\underset{1}{\cancel{5}}} = \frac{1}{\underset{1}{\cancel{2}}} \times \overset{5}{\cancel{10}}$$

$$2p = 5$$

The multiplicative inverse of 2 is $\frac{1}{2}$. So, to isolate the variable, multiply both sides by $\frac{1}{2}$ or divide by 2.

$$2p = 5 \qquad \text{or} \qquad \frac{2p}{2} = \frac{5}{2}$$

$$\frac{1}{2} \times 2p = 5 \times \frac{1}{2} \qquad\qquad p = \frac{5}{2} \text{ or } 2.5$$

$$\frac{1}{\underset{1}{\cancel{2}}} \times \overset{1}{\cancel{2}}p = 5 \times \frac{1}{2}$$

$$p = \frac{5}{2} \text{ or } 2.5$$

Method 2

Write the equation.

$$\frac{p}{5} = \frac{1}{2}$$

Then find the **cross products**. This is a short cut to clearing the denominators.

$$\frac{p}{5} \quad \frac{1}{2}$$

$$2 \times p = 5 \times 1$$

$$2p = 5$$

The multiplicative inverse of 2 is $\frac{1}{2}$. So, to isolate the variable, multiply both sides by $\frac{1}{2}$ or divide by 2.

$$\frac{1}{2} \times 2p = 5 \times \frac{1}{2} \qquad \text{or} \qquad \frac{2p}{2} = \frac{5}{2}$$

$$\frac{1}{\underset{1}{\cancel{2}}} \times \overset{1}{\cancel{2}}p = 5 \times \frac{1}{2} \qquad\qquad p = \frac{5}{2} \text{ or } 2.5$$

$$p = \frac{5}{2} \text{ or } 2.5$$

Note: You can use a calculator to assist you with the calculations.

Key Press	Display
5 × 1 ÷ 2 =	2.5

Example 2: Solve $\frac{3}{a} = \frac{5}{4}$.

Solution

Method 1

First write the equation.

$$\frac{3}{a} = \frac{5}{4}$$

The product of the denominators is $4a$. So, multiply both sides of the equation by $4a$ and simplify.

$$4a \times \frac{3}{a} = \frac{5}{4} \times 4a$$

$$\overset{1}{4\cancel{a}} \times \frac{3}{\underset{1}{\cancel{a}}} = \frac{5}{\underset{1}{\cancel{4}}} \times \overset{1}{\cancel{4}}a$$

$$12 = 5a$$

The multiplicative inverse of 5 is $\frac{1}{5}$. So, to isolate the variable, multiply by $\frac{1}{5}$ or divide by 5.

$$5a = 12 \qquad \text{or} \qquad \frac{5a}{5} = \frac{12}{5}$$

$$\frac{1}{5} \times 5a = \frac{1}{5} \times 12 \qquad\qquad a = \frac{12}{5} \text{ or } 2.4$$

$$a = \frac{12}{5} \text{ or } 2.4$$

Method 2

First write the equation.

$$\frac{3}{a} = \frac{5}{4}$$

Then find the cross products. This is a short cut to clearing the denominators.

$$\frac{3}{a} \;\times\; \frac{5}{4}$$

$$3 \times 4 = 5a$$

$$12 = 5a$$

The multiplicative inverse of 5 is $\frac{1}{5}$. So, to isolate the variable, multiply by $\frac{1}{5}$ or divide by 5.

$$\frac{1}{5} \times 5a = \frac{1}{5} \times 12 \qquad \text{or} \qquad \frac{5a}{5} = \frac{12}{5}$$

$$a = \frac{12}{5} \text{ or } 2.4 \qquad\qquad a = \frac{12}{5} \text{ or } 2.4$$

Note: You can use a calculator to help you with the calculations.

Key Press	Display
3 × 4 ÷ 5 =	2.4

Practice Activity 4

1. Solve each of the following equations by clearing the denominators.

   a. $\frac{n}{5} = \frac{3}{4}$ b. $\frac{2}{n} = \frac{5}{8}$ c. $\frac{x}{3} = \frac{4}{5}$ d. $\frac{3}{r} = \frac{5}{8}$

2. Solve the equations in Question 1 by using the cross-products method.

3. Indicate the keys you would press on a calculator to solve the equations in Question 1.

Turn to the Appendix to check your answers.

Did You Know?

Niels Henrik Abel

Niels Henrik Abel was born in 1802 in Norway. His father, who was poor but well educated, was a minister. His mother was a beautiful woman, and Niels got his own good looks from her. Niels had six brothers and sisters. Even though they sometimes did not have enough to eat, they were a happy bunch and liked to sit around the fire joking and talking together.

When Niels was very young, he found that he liked mathematics. At the age of sixteen he was reading books by the great masters and finding a few mistakes. In his school one teacher often beat the boys, but another teacher was kind to Niels. This teacher saw that he was very bright in mathematics and helped him with his studies.

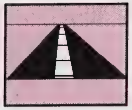
Niels' father died when Niels was eighteen years old. The care for his mother and six brothers and sisters fell on the young man's shoulders. He took up his duties with a smile on his face. He always had a happy way of looking at life.

Somehow he was able to pay his way through university. Then Norway sent him to Germany and France to study. He hoped to meet many famous mathematicians there, but most of them had no time for him. A German by the name of Crelle was one of the few who saw how bright Abel was. He and Abel started one of the greatest mathematical magazines of all time.

Abel died at the early age of twenty-six. Even in his short life he had so many ideas about mathematics that students all over the world still wonder at his thoughts.[1]

[1] The National Council of Teachers of Mathematics for the excerpt from *Mathematical History*.

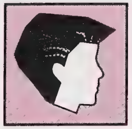
SOLVING EQUATIONS OF THE FORM $ax + b = c$



What Lies Ahead

In this section you will learn these skills.

- using formal procedures to solve equations of the form $ax + b = c$
- solving problems using equations



Working Together

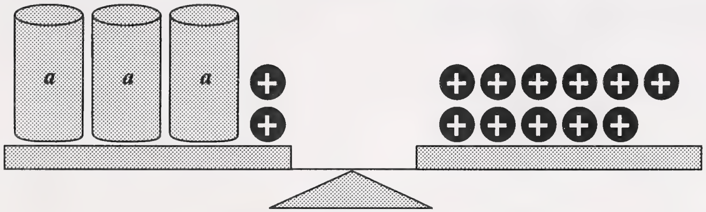
In Section 5 you solved equations using additive inverses. In Section 6 you solved equations using multiplicative inverses.

In this section you will solve equations using both additive inverses and multiplicative inverses.

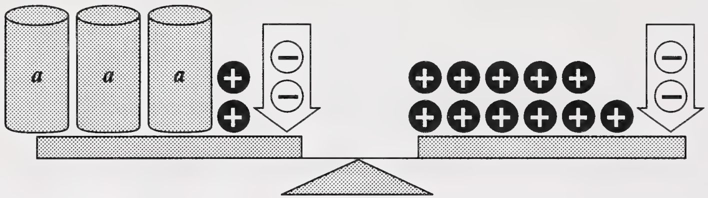
Example: Solve the equation $3a + 2 = 11$.

Solution

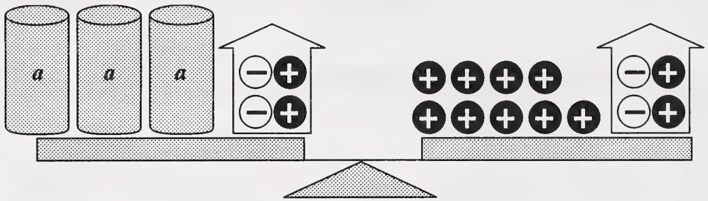
First model the equation.



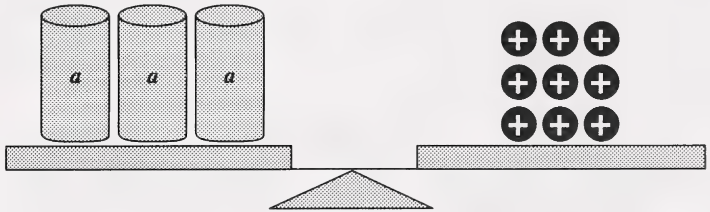
In order to isolate a, add -2 (the additive inverse of $+2$) to each side.



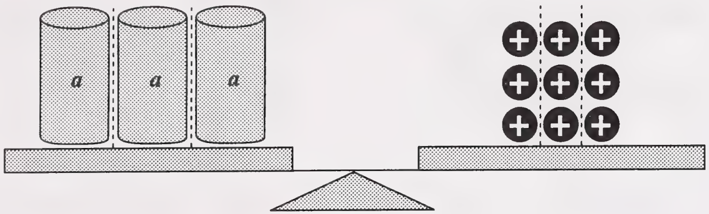
Simplify the equation by removing the zeros. This will not change the balance.



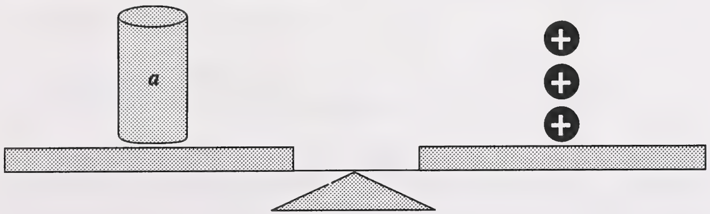
The result is this.



In order to isolate a, divide each side into three groups.



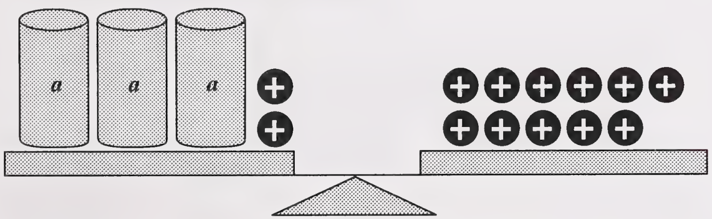
To solve the equation, examine only one group from each side.



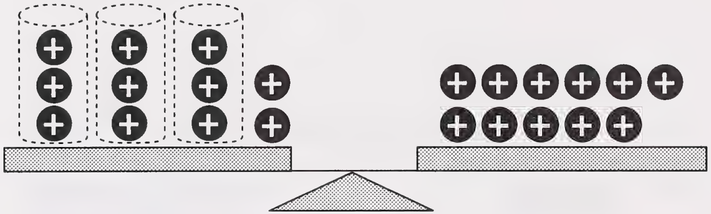
The solution is $a = +3$.

You can verify Example 1 to check your answer.

First model the equation.



Then replace a with 3.

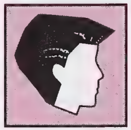


Both sides have a value of $+11$ if $a = 3$. The scale is balanced.

The solution is $a = 3$.

Video Activity

Watch the first segment of the video *MATH MOVES: Equations – Solving With More Than One Step*. It is entitled "Solving Equations: $ax + b = c$".



Working Together

Solving Equations by Using Paper-and-Pencil Methods

You will now learn to solve equations by using a paper-and-pencil method.

Example 1: Solve the equation $2x + 4 = 10$.

Solution

First write the equation.

$$2x + 4 = 10$$

Add -4 (the additive inverse of 4) to both sides of the equation.

$$2x + 4 + (-4) = 10 + (-4)$$

Simplify the left-hand side of the equation by removing the zeros.

$$2x \cancel{+4} + (\cancel{-4}) = 10 + (-4)$$
$$2x =$$

Simplify the right-hand side of the equation.

$$2x = 6$$

To isolate x, multiply both sides by $\frac{1}{2}$ and simplify.

$$\frac{1}{2} \times 2x = \frac{1}{2} \times 6$$

$$\frac{1}{\cancel{2}_1} \times \cancel{2}^1 x = \frac{1}{\cancel{2}_1} \times \cancel{6}^3$$
$$x = 3$$

Verification

LS	RS
$2x + 4$	10
$= 2 \times 3 + 4$	
$= 6 + 4$	
$= 10$	

LS = RS

So, the solution is indeed $x = 3$.

Example 2: Paper-and-Pencil Method

Solve the equation $4m - 1 = -9$.

Solution

First write the equation.

$$4m - 1 = -9$$

Add $+1$ (the additive inverse of -1) to both sides of the equation.

$$4m - 1 + 1 = -9 + 1$$

Simplify the left-hand side of the equation.

$$4m \cancel{-1} \cancel{+1} = -9 + 1$$
$$4m =$$

Simplify the right-hand side of the equation.

$$4m = -8$$

To isolate m, multiply each side by $\frac{1}{4}$ and simplify.

$$\frac{1}{4} \times 4m = \frac{1}{4} \times (-8)$$

$$\frac{1}{\cancel{4}_1} \times \cancel{4}^1 m = -\left(\frac{1}{\cancel{4}_1} \times \cancel{8}^2\right)$$

$$m = -2$$

The solution is $m = -2$.

Verification

LS	RS
$4m - 1$	-9
$= 4m + (-1)$	
$= 4 \times (-2) + (-1)$	
$= -8 + -1$	
$= -9$	

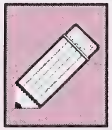
LS = RS

So, the solution is $m = -2$.

Practice Activity 1

Computer Alternative

1. Do Lesson 8 and Lesson 10 on the disk *Pre-Algebra* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

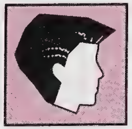


Print Alternative

2. Solve the equations by using a paper-and-pencil method. Be sure to verify your solutions.

   a. $5x + 6 = 31$ **b.** $6x - 3 = 15$ **c.** $8a + 6 = 22$
   d. $3c - 3 = -24$ **e.** $11n + 44 = 0$

Turn to the Appendix to check your answers.



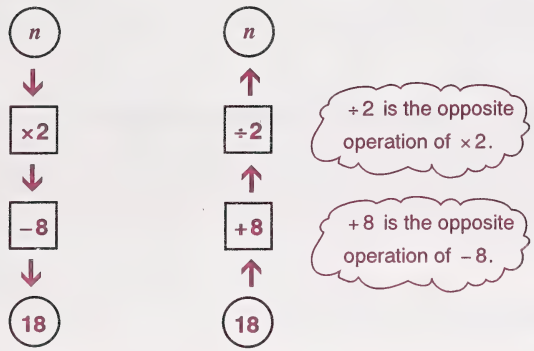
Working Together

Solving Equations by Working Backwards

Another method to solve equations is to use flow charts and inverse flow charts. Flow charts include all steps that are normally used for solving equations, but you do not have to write out all the steps.

Example: Solve $2n - 8 = 18$.

Write the equation in flow chart form. To solve the equation, use an inverse flow chart and work backwards.



So, $n = 13$.

Notice that the flow chart uses the rules for order of operations – BEDMAS. In the example, multiplication is done before subtraction.

The inverse flow chart uses the opposite order. In the example, addition is done before division.

The inverse flow chart undoes what was done in the flow chart.

Verification

LS	RS
$2n - 8$	18
$= 2 \times 13 - 8$	
$= 26 - 8$	
$= 18$	

LS = RS

The solution is $n = 13$.

Practice Activity 2

1. Solve the following equations using flow charts and inverse flow charts.

   a. $3a - 2 = 7$ b. $2b + 1 = -9$ c. $3c - 4 = 5$
   d. $4b - 1 = -9$ e. $2d + 1 = 5$

2. Solve the following equations by using a paper-and-pencil method.

a. $2a+\frac{1}{2}=\frac{3}{4}$ b. $3b-1=8.3$ c. $5c+1.5=-10$

3. Do the puzzle "What Did the Baby Buzzard Say When It Saw an Orange in the Nest?"[1] on the following page.

Turn to the Appendix to check your answers.

Did You Know?

Isaac Newton

Isaac Newton was born on a farm in England on Christmas Day, 1642. His father died before he was born, and Isaac was brought up by his grandmother. He was never very strong but was able to go to a country school.

Young Isaac liked to experiment and to make things. Once he made a little toy mill that could grind wheat into flour. Real mills of his day were sometimes driven by mules or oxen, but Isaac's was driven by a mouse! He also made a wooden clock that worked by water.

Isaac's uncle saw that the boy was very bright and thought he should go to college. Fortunately, his family was able to send him. There, at the age of eighteen, he studied chemistry at first, but he soon began to get interested in mathematics. Isaac read Euclid's *Elements*, which he found to be much too easy for him. The he read the book *Geometry*, by Descartes, a mathematician whose work had been done about the time Isaac was born. This he found more difficult. Next he read works by a few other great mathematicians.

Isaac Newton was so bright that after these readings, he was able to start doing all sorts of hard mathematical problems by himself. When he was twenty-three years old, his college had to close because of the plague. While Newton was at home that year, he worked on the calculus, the subject for which he is famous. He also began his great study of the way the earth and moon move. He decided that they move as they do because of what he called "gravity." Gravity is the force of attraction that exists between all bodies in the universe. It is the force that makes a ball thrown in the air fall back to the earth. It is the force that keeps the earth going around the sun instead of flying off in space. Newton used his idea of gravity to explain a lot of things about the sun, planets, and stars.

When Newton had a problem to think about, he put everything else out of his mind. Once he was thinking about a problem and walking his horse up a hill at the same time. The horse somehow slipped out ot the bridle and ran away. Newton didn't know the horse was gone until, at the top of the hill, he tried to jump on the horse!

Although Newton was one of the greatest mathematicians of all time, he was a very modest man. He felt he owed much to those mathematicians who lived before him. This is what he meant when he said, "If I have seen farther than others, it is because I have stood on the shoulders of giants."

Newton's mathematical ideas are among the most difficult ever thought about and also among the most useful. Modern science owes much to Newton. Many of the things we enjoy today were made possible by Newton's mathematics.[2]

[1] 1989 Creative Publications for excerpt from *Algebra with Pizzazz*.

[2] The National Council of Teachers of Mathematics for the excerpt from *Mathematical History*.

WHAT DID THE BABY BUZZARD SAY WHEN IT SAW AN ORANGE IN THE NEST?

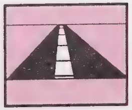
Solve each problem below. Find your answer in the answer column and notice the letter next to it. Write this letter in each box that contains the number of that problem.

1. Two more than 5 times a number is 77. Find the number.
2. Five more than one-third of a number is -2. Find the number.
3. Nine less than one-fourth of a number is 6. Find the number.
4. Sixteen increased by twice a number is -56. Find the number.
5. Twelve decreased by 8 times a number is 36. Find the number.
6. One-eighth of a number, increased by 20, is 32. Find the number.
7. Twenty-five decreased by one-fifth of a number is 18. Find the number.
8. Nine times a number, diminished by 4, is 95. Find the number.
9. The length of a rectangle is 50 m. This is 6 m more than twice the width. Find the width.
10. Grandpa Schmidt is 75 years old. This is 9 years less than seven times the age of Junior Schmidt. How old is Junior?
11. Bill's weight is 48 kg. This is 10 kg more than one-half of his father's weight. What is his father's weight?
12. A medium orange has 70 calories. This is 10 calories less than one-fourth of the calories in a Sugar Krunchy. How many calories are in a Sugar Krunchy?
13. The length of a couch is 200 cm. This is 16 cm less than 3 times the width of a matching chair. How wide is the chair?

12	5	5	8	11	7	7	1	2	5	10	11	4	9	2	13	11	13	11	12	11	6	3

G	22
D	60
M	72
I	96
H	15
S	74
T	35
L	320
N	-36
R	12
E	-21
F	13
K	11
C	342
O	-3
A	76

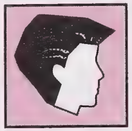
SOLVING EQUATIONS OF THE FORMS $ax + bx = c$ AND $a(x + b) = c$



What Lies Ahead

In this section you will learn these skills.

- using formal procedures to solve equations of the form $ax + bx = c$ and $a(x + b) = c$
- solving problems using equations



Working Together

In this section you will look at equations that must be simplified before you solve them.

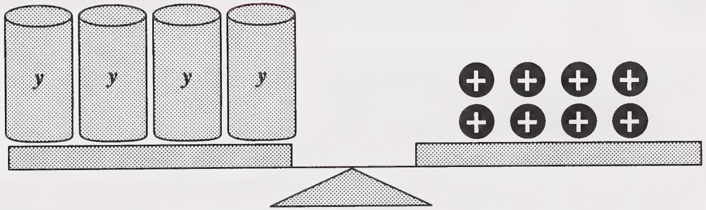
Collecting Like Terms

Some equations require you to collect like terms. Look at the following examples.

Example 1: Solve the equation $2y + 2y = 8$.

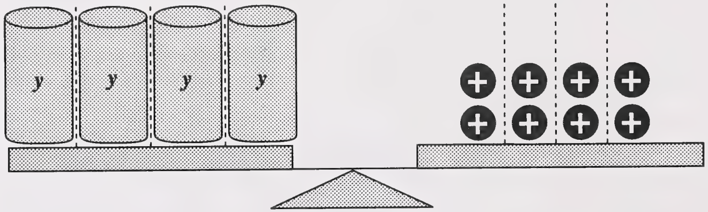
Solution

First model the equation.

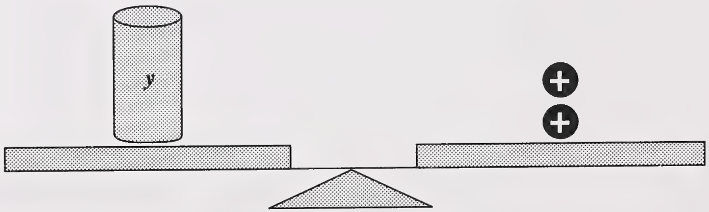


The model shows that $2y + 2y = 4y$.

In order to isolate y, divide each side into four groups.



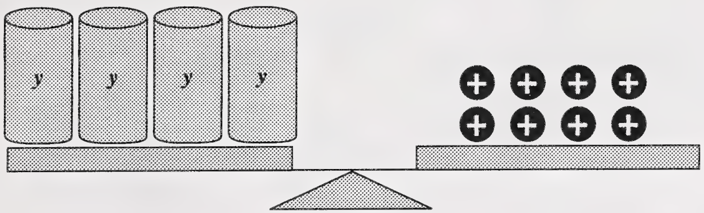
To solve the equation, examine only one group from each side.



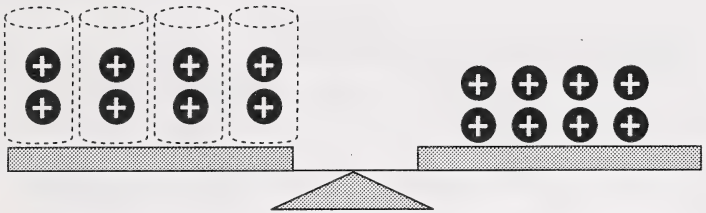
The solution is $y = 2$.

Verifying Example 1

To verify the solution of $2y + 2y = 8$, first model the equation.



Then replace y with 2.



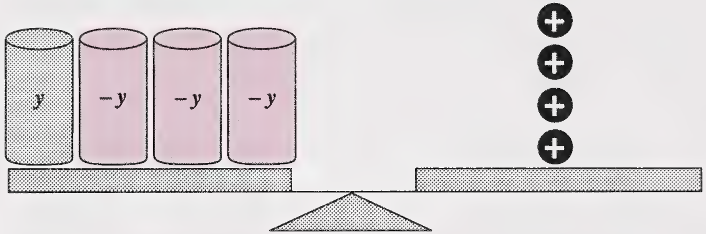
Both sides have a value of $+8$ if $y = 2$. The scale is balanced.

The solution is $y = 2$.

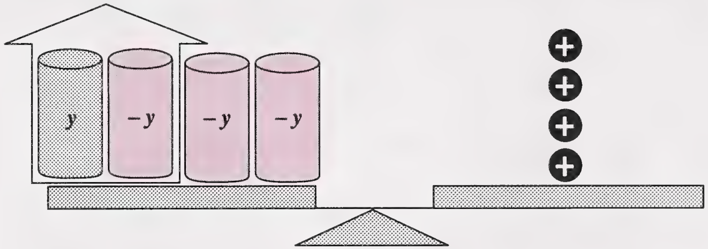
Example 2: Solve $y - 3y = 4$.

Solution

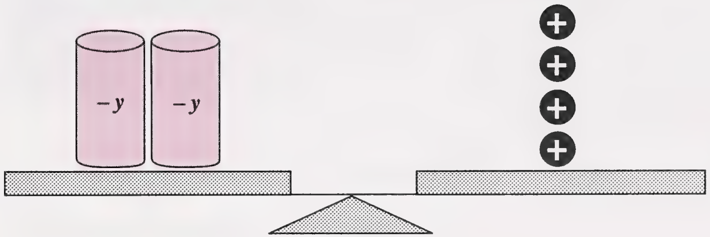
First model the equation.



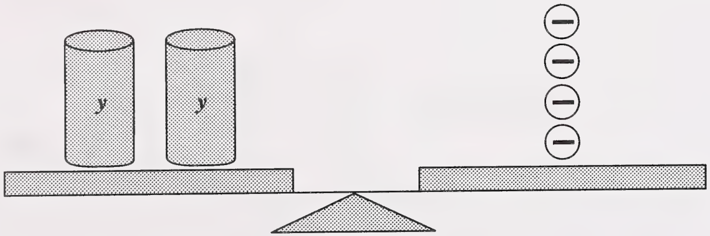
Simplify the equation by removing the zeros.



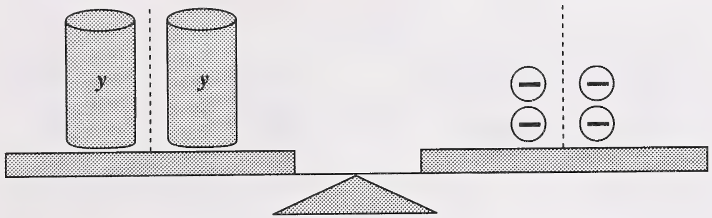
The result is this.



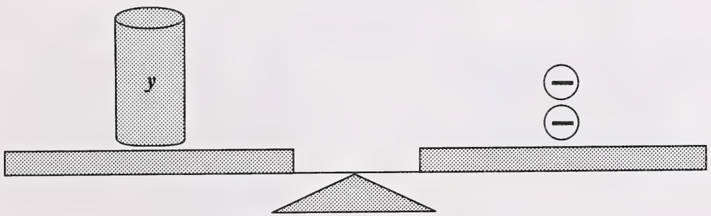
You want to solve for y, so replace each side of the equation with its inverse.



In order to isolate the variable, divide each side into two groups.



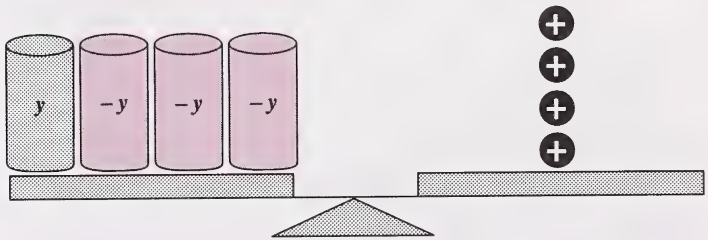
To solve the equation, examine only one group from each side.



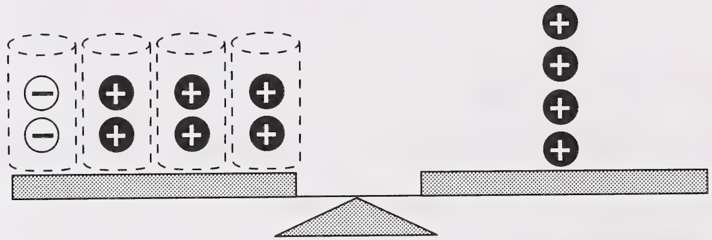
The solution is $y = -2$.

Verifying Example 2

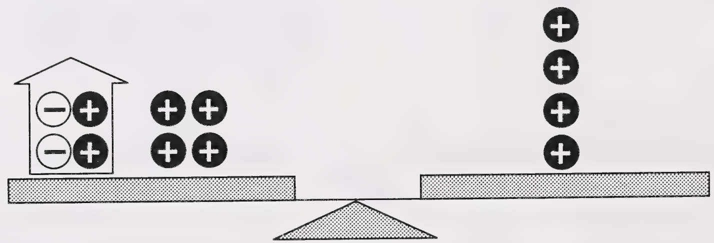
To verify the solution of $y - 3y = 4$, first model the equation.



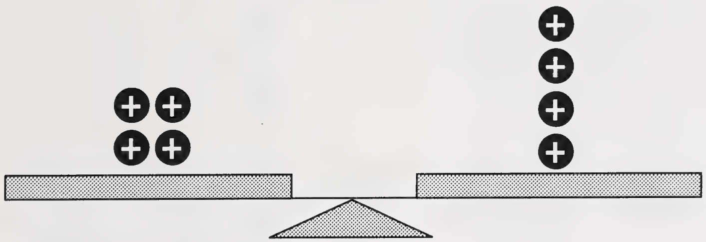
If $y = -2$, $-y = 2$. Replace y with -2 and $-y$ with 2.



Remove the zeros.



This is the result.

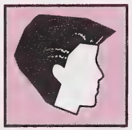


Both sides have a value of $+4$ if $y = -2$. The scale is balanced.

The solution is $y = -2$.

Video Activity

Watch the second segment of the video *MATH MOVES: Equations – Solving With More Than One Step*. This segment is entitled "Solving Equations: $ax + bx = c$". Use the cut-out scale, counters, and cylinders from the Appendix to do the video assignment.



Working Together

Removing Brackets

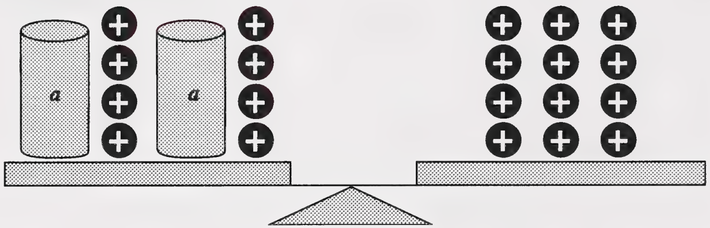
Some equations require you to remove brackets. Look at the following example.

Example: Solve the equation $2(a + 4) = 12$.

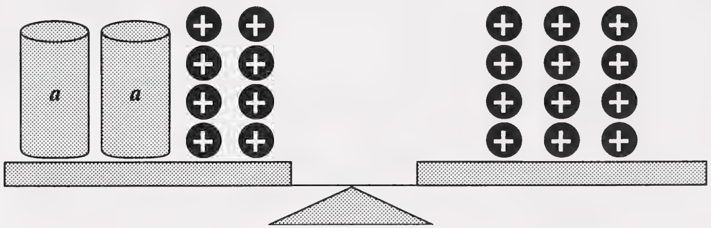
Solution

First model the equation.

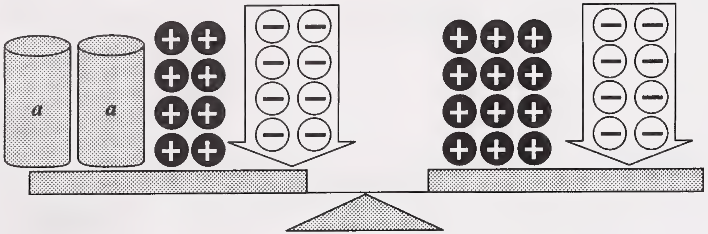
$2(a + 4)$ means two groups of $a + 4$.



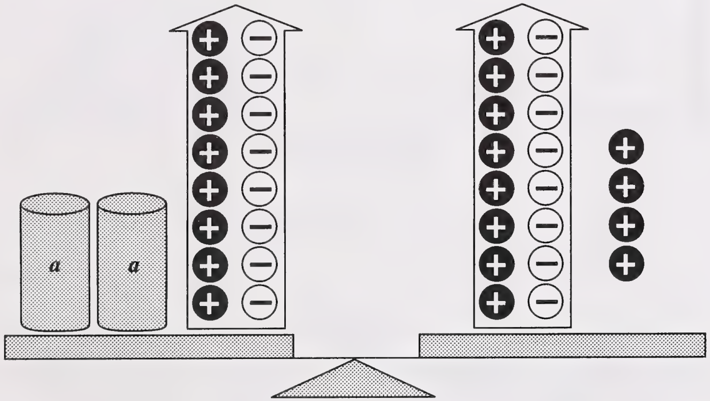
The model shows that $2(a + 4) = 2a + 8$.



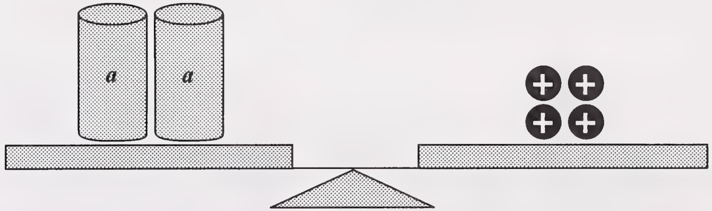
The additive inverse of 8 is -8. So, add -8 to each side of the equation.



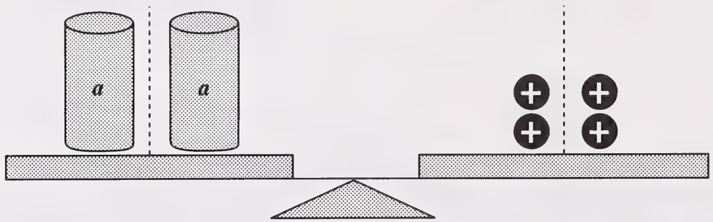
Simplify the equation by removing the zeros. This will not change the balance.



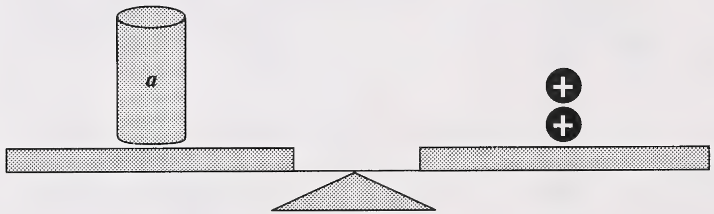
The result is this.



In order to isolate the variable, divide each side into two groups.

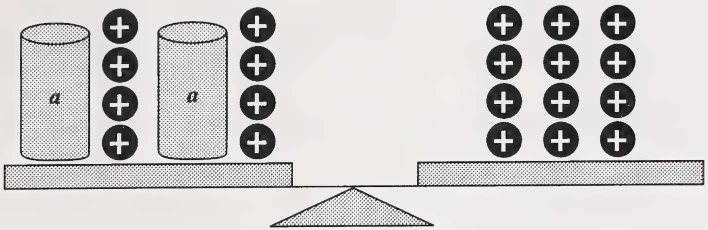


To solve the equation, examine only one group from each side.

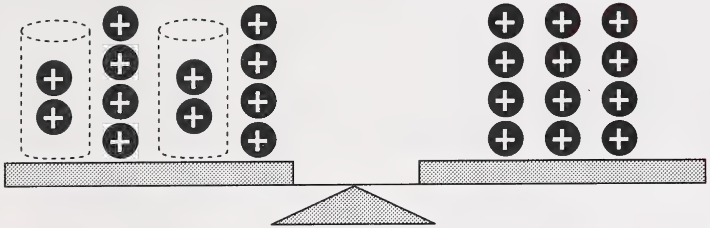


The solution is $a = +2$.

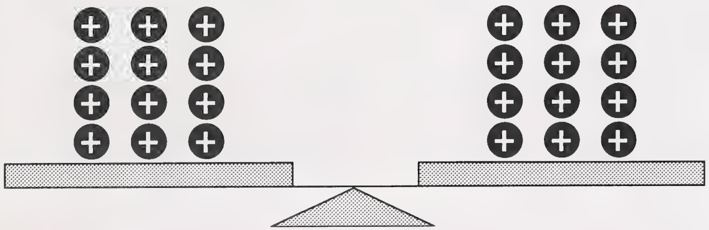
To verify the solution, first model the equation.



Then replace a with 2.



This is the result.

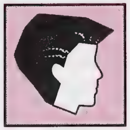


Both sides have a value of 12 if $a = 2$. The scale is balanced.

The solution is $a = 2$.

Video Activity

Watch the fourth segment of the video *MATH MOVES: Equations – Solving With More Than One Step.* The segment is entitled "Solving Equations: $a(x+b)=c$". Use the cut-out scale, counters, and cylinders from the Appendix to do the video assignment.



Working Together

Solving Equations by Using Paper-and-Pencil Methods

You will now learn to solve equations by using a paper-and-pencil method.

Example 1: Solve $3k + 2k = -15$.

Solution

First write the equation.

$$3k + 2k = -15$$

Then collect the like terms.

$$5k = -15$$

The multiplicative inverse of 5 is $\frac{1}{5}$. So, to isolate k, multiply both sides by $\frac{1}{5}$.

$$\frac{1}{5}\times 5k=\frac{1}{5}\times(-15)$$

Simplify each side of the equation.

$$\frac{1}{5}\times 5k=-\left(\frac{1}{5}\times 15\right)$$
$$\frac{1}{\cancel{5}_1}\times \cancel{5}^1k=-\left(\frac{1}{\cancel{5}_1}\times \cancel{15}^3\right)$$
$$k=-3$$

The solution is $k=-3$.

Verification

LS	RS
$3k+2k$	-15
$=3\times(-3)+2\times(-3)$	
$=-9+-6$	
$=-15$	

LS = RS

So, the solution is $k=-3$.

Example 2: Solve $2n-3n=-6$.

Solution

First write the equation.

$$2n-3n=-6$$

Next combine like terms.

$$2n+(-3n)=-6$$
$$-1n=-6$$

Replace each side with its inverse.

$$n=6$$

Verification

LS	RS
$2n-3n$	-6
$=2\times 6-3\times 6$	
$=2\times 6+(-3)\times 6$	
$=12+(-18)$	
$=-6$	

LS = RS

Both sides of the equation have a value of -6. The scale is balanced.

So, the solution is $n=6$.

Practice Activity 1

1. Solve and verify the following equations using paper-and-pencil methods.

   a. $2x+4x=12$ **b.** $6y-2y=-8$ **c.** $a-3a=6$

2. Solve and verify the following equations using paper-and-pencil methods.

   a. $3(y-2)=6$ **b.** $2(x+1)=-6$ **c.** $3(7-4x)=29$

3. Solve the following equations. Place the letter that goes with each equation below the solution in the boxes below.

A	$8y-5y=30$	H	$4m+3m=84$
L	$4a+2a+3a=18$	O	$8x+2x-2=28$
V	$3x+5x-2x=36$	I	$12y-11y=8-3$
T	$9b-4b+2b=77$	M	$10a-5a-7+5=33$
E	$10c+4c-6c=64$		

5	4	2	3	6	8	9	7	10	11	12

Turn to the Appendix to check your answers.

Working Together

Some word problems involve more than one unknown number or quantity.

Example 1

The second of two numbers is 7 times the first. Their sum is 32. Find the numbers.

Solution

Write statements for the unknowns. Use only one variable.

Let n be the first number.
Let $7n$ be the second number.

The second of two numbers is seven times the first.

Write an equation.

$$n+7n=32 \leftarrow \text{Their sum is 32.}$$

Solve the equation.

$$n+7n=32$$
$$8n=32$$
$$\frac{1}{\cancel{8}_1}\times\cancel{8}^1n=\frac{1}{\cancel{8}_1}\times\cancel{32}^4$$
$$n=4$$

The first number is 4.

Evaluate the second number.

$$7n = 7 \times 4$$
$$= 28$$

The second number is 28.

Looking Back

Reread the problem and be sure your answer is true.

Is 28 seven times 4? Yes.
Is the sum of 4 and $28 = 32$? Yes.

Example 2

The greater of two numbers is 8 more than 5 times the smaller number. Their difference is 48. Find the numbers.

Solution

Write statements for the unknowns.
Use only one variable.

Let x be the lesser number.
Let $5x + 8$ be the greater number.

The greater of the two numbers is 8 more than 5 times the smaller number.

Write an equation.

$$5x + 8 - x = 48$$

The difference of the two numbers is 48.

$$\begin{aligned} 4x + \cancel{8} &= 48 \\ \cancel{-8} & \quad -8 \\ \hline 4x &= 40 \\ x &= 10 \end{aligned}$$

The lesser number is 10. Evaluate the second number.

$$\begin{aligned} 5x + 8 &= 5 \times 10 + 8 \\ &= 50 + 8 \\ &= 58 \end{aligned}$$

The greater number is 58.

Looking Back

Reread the problem and be sure your answer is true.

Is 58 8 more than 5 times 10? Yes.
Is $58 - 10 = 48$? Yes.

Practice Activity 2

Do the following puzzle "Why Is a Yo-Yo Like Waking Up at 5 a.m?"[1]

Turn to the Appendix to check your answers.

[1] 1982 Creative Publications for excerpt from *Pre-Algebra with Pizzazz.*

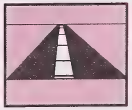
WHY IS A YO-YO LIKE WAKING UP AT 5 A.M.?

Solve each problem and find your answers at the bottom of the page. Shade out the letter above each correct answer. When you finish, the answer to the title question will remain!

1. The second of two numbers is 4 times the first. Their sum is 45. Find the numbers.
2. The greater of two numbers is 3 times the lesser. Their sum is 44. Find the numbers.
3. The second of two numbers is 7 more than the first. Their sum is 47. Find the numbers.
4. The greater of two numbers is 10 more than the lesser. Their sum is 38. Find the numbers.
5. The sum of two numbers is 31. The first is 5 less than the second. Find the numbers.
6. The second of two numbers is 1 more than twice the first. Their sum is 25. Find the numbers.
7. The greater of two numbers is 8 more than 5 times the lesser. Their sum is 68. Find the numbers.
8. The second of two numbers is 3 less than twice the first. Their sum is 42. Find the numbers.
9. Find two numbers whose sum is 33, if the second is 2 less than 4 times the first.
10. A basketball player shot 70 times. The number of missed shots was 6 more than the number of baskets. How many baskets did the player make?
11. The entertainment portion of a 60-min TV program lasted 4 times as long as the advertising portion. How many minutes of advertising were there?

G	T	I	H	E	T	U	S	T	O	W	P	I	N	R	L	E	N	Y	O
14, 24	32	8, 25	20, 27	10, 58	14, 26	9, 36	15, 25	29	12	11	7, 26	10, 40	8, 17	15, 20	12, 56	11, 33	13, 18	9, 15	15, 27

SOLVING EQUATIONS OF THE FORMS $ax + b = cx$ AND $ax + b = cx + d$



What Lies Ahead

In this section you will learn these skills.

- using formal procedures to solve equations of the form $ax + b = cx$ and $ax + b = cx + d$
- solving problems using equations

Working Together

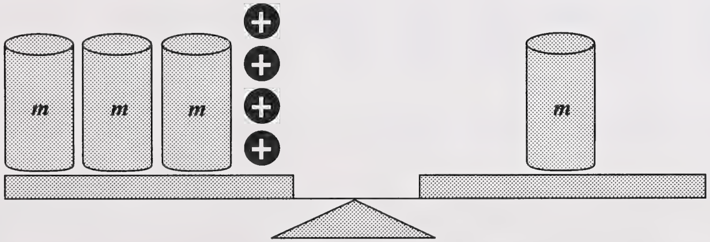
This section of the module looks at equations with variables on both sides of the equation.

Sometimes there is a variable on one side of the equation and a variable and a constant on the other side of the equation.

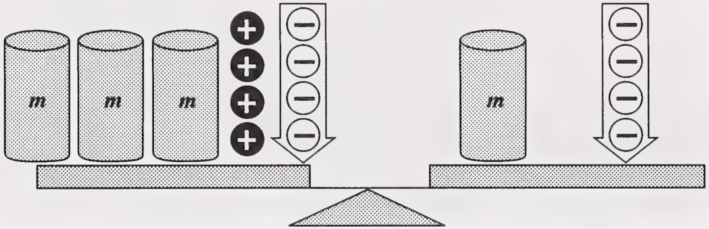
Example: Solve the equation $3m + 4 = m$.

Solution

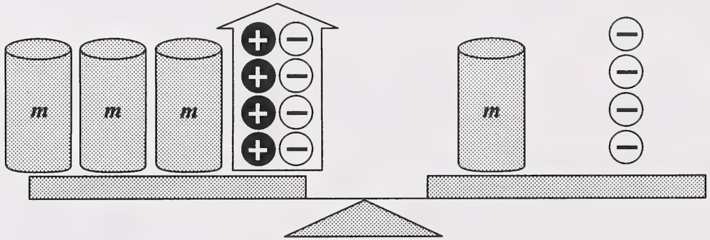
Model the equation.



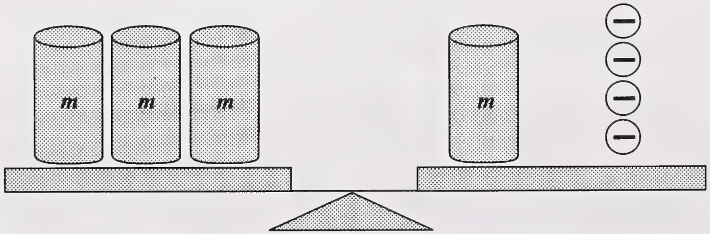
To isolate $3m$ on the left-hand side, add -4 to each side. -4 is the additive inverse of $+4$.



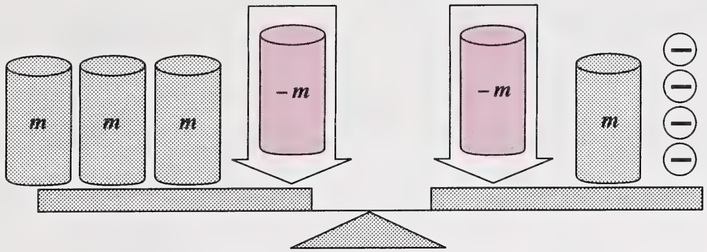
Simplify by removing the zeros.



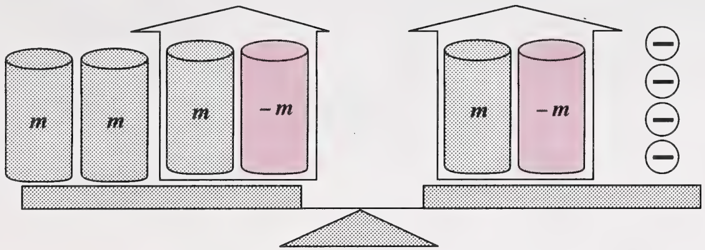
The result is this.



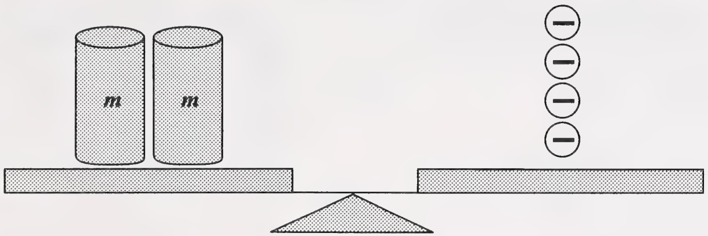
To isolate -4 on the right-hand side, add $-m$ to each side. $-m$ is the inverse of m.



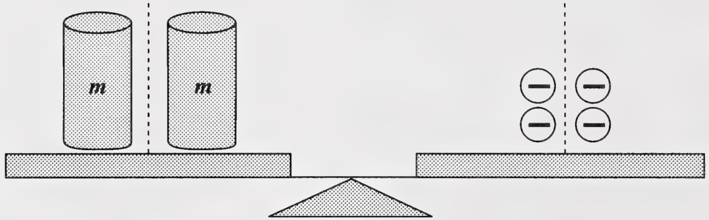
Remove the zeros.



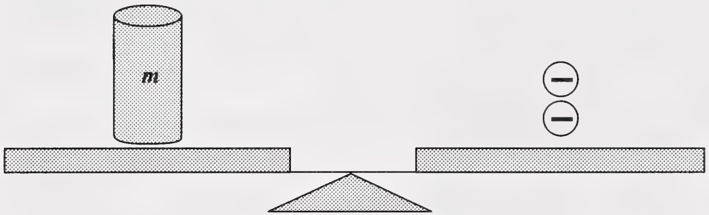
The result is this.



To isolate m, divide each side into two groups.

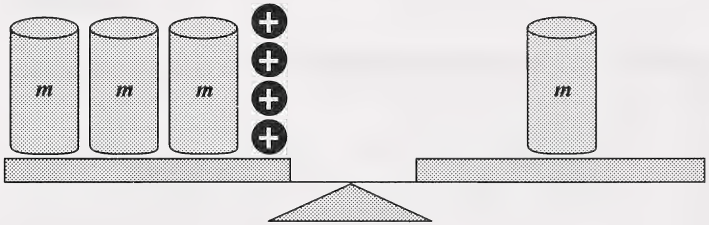


To solve the equation, examine only one group from each side.

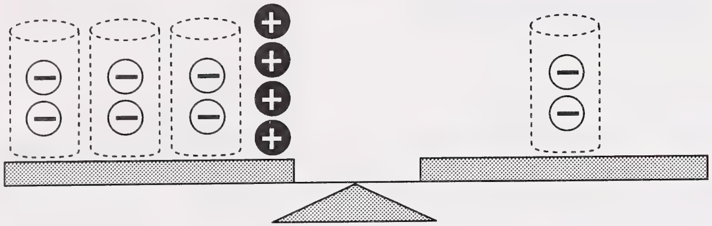


So, the solution is $m = -2$.

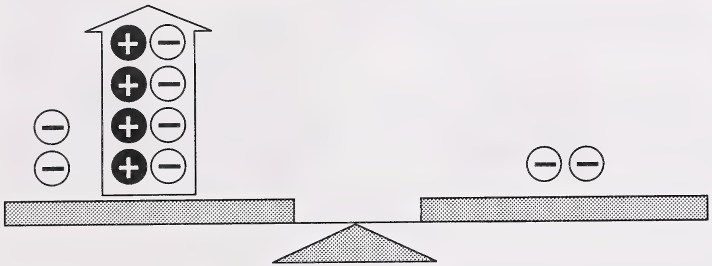
To verify the solution, model the equation.



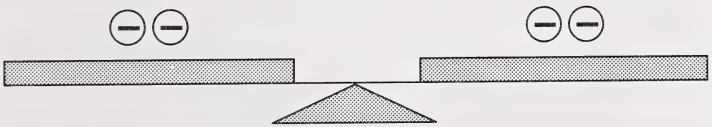
Replace m with -2.



Remove the zeros.



The result is this.



The value of each side of the equation is -2. The scale is balanced.

So, the solution is $m = -2$.

Video Activity

Watch the third segment of the video *MATH MOVES: Equations – Solving With More Than One Step*. The segment is entitled "Solving Equations : $ax + b = cx$". Use the cut-out scale, counters, and cylinders in the Appendix to do the video assignment.

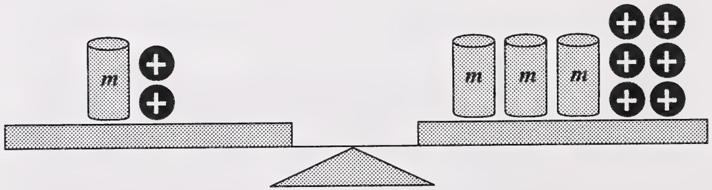
Working Together

Sometimes there is a variable and a constant on each side of the equation.

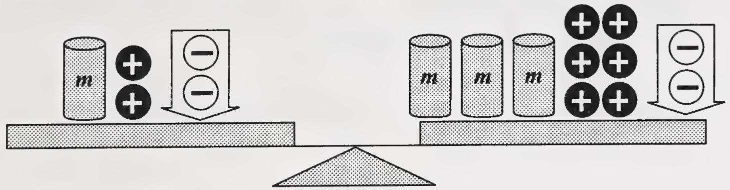
Example: Solve the equation $m + 2 = 3m + 6$.

Solution

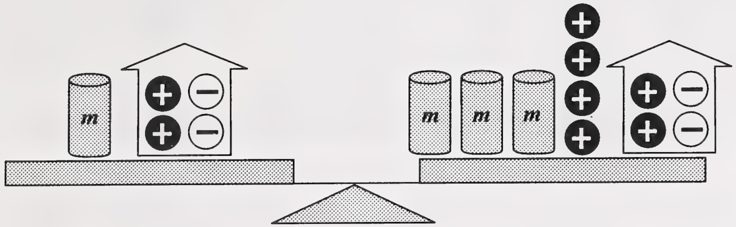
Model the equation.



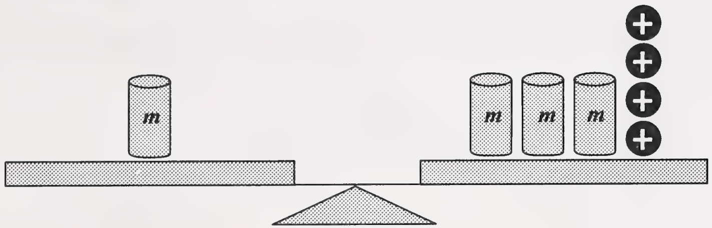
To isolate m on the left-hand side, add -2 to each side.



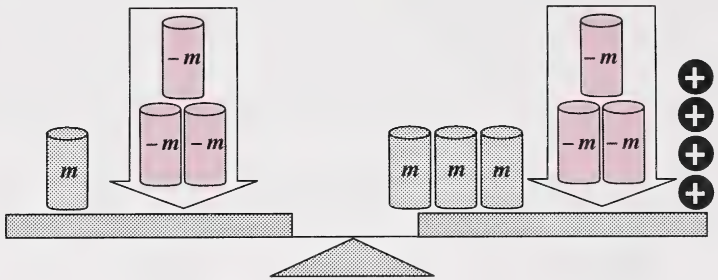
Remove the zeros.



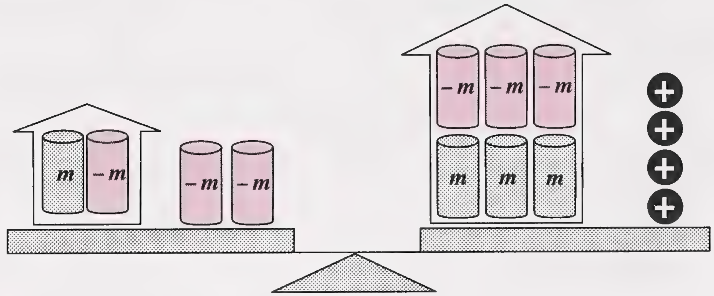
The result is this.



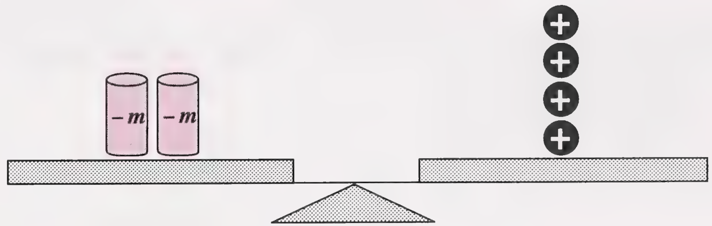
To isolate $+4$ on the right-hand side, add $-3m$ to each side.



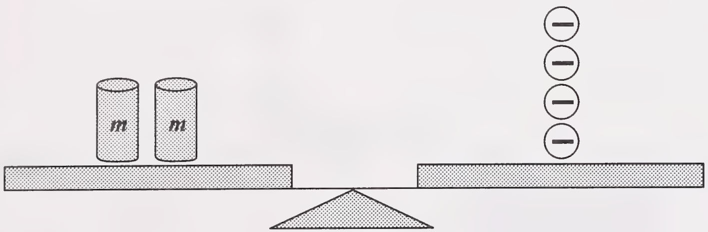
Remove the zeros.



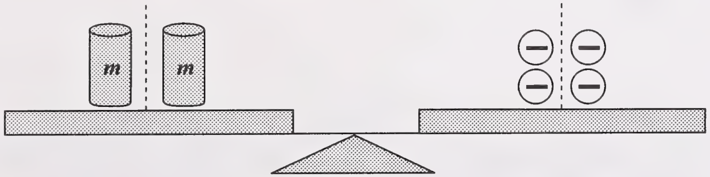
The result is this.



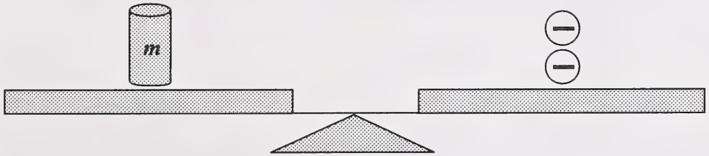
Replace each side with its inverse.



Divide each side into two groups.

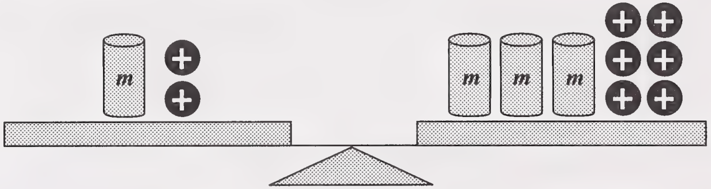


To solve the equation, examine only one group from each side.

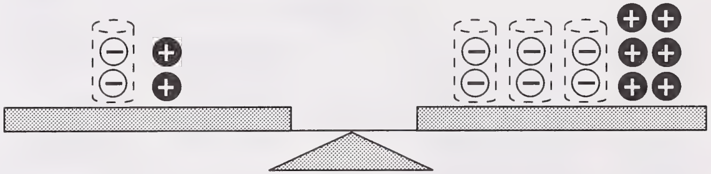


So, the solution is $m = -2$.

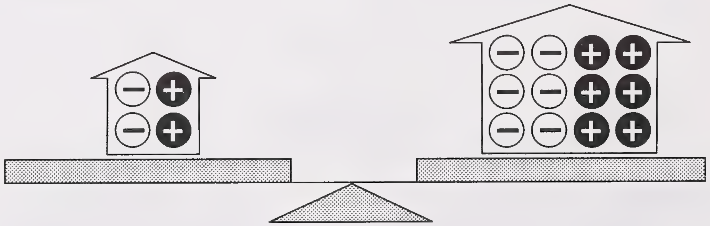
To verify the solution, model the equation.



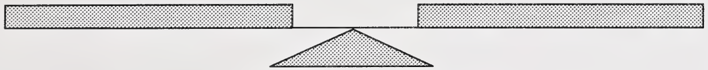
Replace m with -2.



Remove the zeros.



The result is this.



The value of each side of the equation is zero. The scale is balanced.

So, the solution is $m = -2$.

Video Activity

Watch the fifth segment of the video *MATH MOVES: Equations – Solving With More Than One Step*. It is entitled "Solving Equations: $ax + b = cx + d$". Use the cut-out scale, counters, and cylinder from the Appendix to do the video assignment.

Working Together

Solving Equations by Using Paper-and-Pencil Methods

You will now learn to solve equations by using a paper-and-pencil method.

Example 1: Solve $3x - 2 = x$.

Solution

Write the equation.

$$3x - 2 = x$$

To isolate $3x$ on the left-hand side of the equation, add 2 to each side and simplify the equation.

$$\begin{aligned} 3x \cancel{-2} &= x \\ \cancel{+2} & \quad +2 \\ \hline 3x &= x + 2 \end{aligned}$$

To isolate $+2$ on the right-hand side, add $-x$ to each side of the equation and simplify the equation.

$$\begin{aligned} 3x &= \cancel{x} + 2 \\ -x &= \cancel{-x} \\ \hline 2x &= 2 \end{aligned}$$

To isolate the variable, multiply each side by $\frac{1}{2}$ and simplify.

$$\frac{1}{2} \times 2x = \frac{1}{2} \times 2$$

$$\frac{1}{\cancel{2}_1} \times \cancel{2}^1 x = \frac{1}{\cancel{2}_1} \times \cancel{2}^1$$

$$x = 1$$

The solution is $x = 1$.

Verification

LS	RS
$3x-2$	1
$=3\times1-2$	
$=3-2$	
$=1$	

LS = RS

The solution is $x=1$.

Example 2: Solve $3m+3=2m+9$.

Solution

Write the equation.

$$3m+3=2m+9$$

To isolate $3m$ on the left-hand side of the equation, add -3 to each side and simplify the equation.

$$\begin{aligned} 3m\cancel{+3} &= 2m+9 \\ \cancel{-3} &\quad -3 \\ \hline 3m &= 2m+6 \end{aligned}$$

To isolate $+6$ on the right-hand side, add $-2m$ to each side and simplify the equation.

$$\begin{aligned} 3m &= \cancel{2m}+6 \\ -2m &\quad -\cancel{2m} \\ \hline m &= 6 \end{aligned}$$

So, the solution is $m=6$.

Verification

LS	RS
$3m+3$	$2m+9$
$=3\times6+3$	$=2\times6+9$
$=18+3$	$=12+9$
$=21$	$=21$

LS = RS

The solution $m=6$ is true.

Practice Activity

1. Solve the following equations using paper-and-pencil methods.

   a. $5n-8=3n$ **b.** $10-n=-6n$

   c. $3x+9=2x-5$ **d.** $4-8a=2a-66$

   e. $3+2a=-5a-11$

2. Do the puzzle "Books Never Written"[1] on the following page.

Turn to the Appendix to check your answers.

[1] 1982 Creative Publications for excerpt from *Pre-Algebra with Pizzazz.*

BOOKS NEVER WRITTEN

Tragedy on the Cliff by ________ ■ ________

4 −7 3 4 4 −11 6 12 −2 4 −5

Mystery of the Creaking Door by ________ ■ ________

−5 −15 10 −9 1 8 −7 −11 −4 4 10

P.S. by ________ ■ ________

−1 6 −1 3 −7 −11 4 2 12 12 −5 4

Above are the titles of three "Books Never Written." To decode the names of their authors, follow these directions.

Solve any problem below and find the answer in the code at the top of the page. Each time it appears, write the letter of that problem above it. Keep working and you will decode the names of all three authors.

(L) Eight less than 7 times a number is the same as 4 more than 3 times the number. Find the number.

(V) Four more than 6 times a number is the same as 9 times the number increased by 10. Find the number.

(I) Four times a number is the same as 14 less than twice the number. Find the number.

(Y) One more than 8 times a number is the same as 12 times the number decreased by 3. Find the number.

(G) Twelve less than a number is the same as 5 times the number increased by 4. Find the number.

(D) Twenty decreased by 2 times a number is the same as 10 less than 3 times the number. Find the number.

(M) Two more than a number is the same as 16 decreased by 6 times the number. Find the number.

(R) Four times a number decreased by 25 is the same as 9 times the number. Find the number.

(T) Twice a number plus 6 times the number is −72. Find the number.

(H) A number plus 5 more than 3 times the number is 37. Find the number.

(A) Eleven diminished by 5 times a number is the same as 4 times the number increased by 20. Find the number.

(S) Eight times a number is the same as 90 decreased by the number. Find the number.

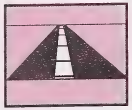
(U) Nine more than 3 times a number is the same as 6 less than twice the number. Find the number.

(O) One increased by 7 times a number is the same as 5 times the number plus 25. Find the number.

(E) Twenty-eight decreased by 6 times a number is the same as the number. Find the number.

(N) Sixteen diminished by 8 times a number is the same as 5 diminished by 9 times the number. Find the number.

SOLVING EQUATIONS OF THE FORMS $x^2 + a = b$, $ax^2 = b$, $\sqrt{x} + a = b$, AND $a\sqrt{x} = b$



What Lies Ahead

In this section you will learn this skill.

- using formal procedures to solve equations of the form $x^2 + a = b$, $ax^2 = b$, $\sqrt{x} + a = b$, and $a\sqrt{x} = b$

Working Together

In this module you have learned this rule to maintain equality in an equation: when you perform an operation on one side of an equation, you must perform the same operation on the other side of the equation. This rule applies to squaring and taking the square root, as well as adding, subtracting, multiplying, and dividing.

Example 1: Solve $a^2 - 3 = 13$.

Solution

Add 3 to each side of the equation.

$$\begin{array}{rcl} a^2 - 3 & = & 13 \\ +3 & & +3 \\ \hline a^2 & = & 16 \end{array}$$

Take the square root of each side.

$$\sqrt{a^2} = \sqrt{16}$$
$$a = 4$$

The solution is $a = 4$.

To verify the solution, use a chart.

LS	RS
$a^2 - 3$	13
$= 4^2 - 3$	
$= 16 - 3$	
$= 13$	

LS = RS

So, $a = 4$ is true.

Example 2: Solve $3r^2 = 300$.

Solution

Multiply each side by $\frac{1}{3}$.

$$3r^2 = 300$$
$$\frac{1}{\cancel{3}_1} \times \cancel{3}^1 r^2 = \frac{1}{\cancel{3}_1} \times \cancel{300}^{100}$$
$$r^2 = 100$$

Take the square root of each side.

$$\begin{aligned}\sqrt{r^2} &= \sqrt{100}\\ r &= 10\end{aligned}$$

To verify the solution, use a chart.

LS	RS
$3r^2$	300
$= 3 \times 10^2$	
$= 3 \times 100$	
$= 300$	

LS = RS

So, $r = 10$ is true.

Example 3: Solve $\sqrt{m} + 3 = 5$.

Solution

Add -3 to each side of the equation.

$$\begin{aligned}\sqrt{m} + 3 &= 5\\ -3 \quad &\; -3\\ \hline \sqrt{m} \quad &= 2\end{aligned}$$

Square each side of the equation.

$$\begin{aligned}\left(\sqrt{m}\right)^2 &= 2^2\\ m &= 4\end{aligned}$$

To verify the solution, use a chart.

LS	RS
$\sqrt{m} + 3$	5
$= \sqrt{4} + 3$	
$= 2 + 3$	
$= 5$	

LS = RS

So, $m = 4$ is true.

Example 4: Solve $3\sqrt{a} = 6$.

Solution

Multiply each side of the equation by $\frac{1}{3}$.

$$\begin{aligned}3\sqrt{a} &= 6\\ \frac{1}{\cancel{3}_1} \times \cancel{3}^1\sqrt{a} &= \frac{1}{\cancel{3}_1} \times \cancel{6}^2\\ \sqrt{a} &= 2\end{aligned}$$

Square each side of the equation.

$$\left(\sqrt{a}\right)^2 = 2^2$$
$$a = 4$$

To verify the solution, use a chart.

LS	RS
$3\sqrt{a}$	6
$= 3\sqrt{4}$	
$= 3 \times 2$	
$= 6$	

LS = RS

So, $a = 4$ is true.

Practice Activity

1. Solve the following equations.

   a. $x^2 + 1 = 10$
   b. $r^2 - 1 = 24$
   c. $3t^2 = 108$
   d. $-2n^2 = -128$
   e. $\sqrt{s} + 2 = 8$
   f. $5\sqrt{n} = 10$

2. Solve the following problems.

   a. The square root of a number, plus three, is twelve. What is the number?

   b. The square of a number, less eight, is seventeen. What is the number?

   c. Ten times the square of a number is forty. What is the number?

   d. Three times the square root of a number is fifteen. What is the number?

Turn to the Appendix to check your answers.

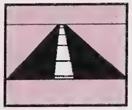
Did You Know?

Sophie Germain

Sophie Germain was a Frenchwoman who lived around 1800. She became a mathematician who was well known in Europe. She studied the patterns that sand makes on a drum when it is hit, and she studied patterns with numbers.[1]

[1] The National Council of Teachers of Mathematics for the excerpt from *Mathematical History*.

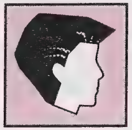
MORE THAN ONE WAY TO GO



What Lies Ahead

In this section you will review these skills.

- simplifying a problem
- using guess-check-revise to solve problems
- using tables to organize information
- using diagrams to solve a problem
- using equations to solve a problem

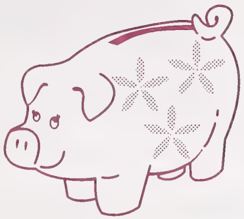


Working Together

In Module 1 you learned several strategies for solving problems. As you have discovered in this module, writing equations is another problem-solving strategy. Problems can also be solved by other methods.

Example 1

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is \$8.50, how many coins of each type does she have ?



Solution

To help you understand the problem, you may wish to simplify it. Here is a similar but simpler problem.

Simpler Problem

Yvonne has 10 nickels and 3 dimes in her bank. What is the total value of the coins?

Solution to the Simpler Problem

Since it is easier to work with whole numbers than decimal numbers, express the value of the coins in cents. You may wish to use a table to organize the data.

Type of Coin	Number of Coins	Value of One Coin (in cents)
Dimes	3	10
Nickels	10	5

$$\begin{aligned}(10 \times 5) + (3 \times 10) &= 50 + 30 \\ &= 80\end{aligned}$$

Yvonne has 80 cents, or \$0.80, in her piggy bank.

Solving the simpler problem gives you a better understanding of the original problem – the total value of the coins is the sum of the product of the number of nickels and the value of a nickel and the product of the number of dimes and the value of a dime.

There are several ways to solve the problem once you understand it.

Method 1: Guess-Check-Revise Method

You can make a guess and test to see if the guess is correct.

Guess 1

Type of Coin	Number of Coins	Value in Cents
Dimes	10	100
Nickels	30	150
Total		250

The total of 250 cents, or $2.50, is too low, so there are more nickels and dimes.

Guess 2

Type of Coin	Number of Coins	Value in Cents
Dimes	20	200
Nickels	40	200
Total		400

The total of 400 cents, or $4.00, is still too low, so there are more nickels and dimes.

Guess 3

Type of Coin	Number of Coins	Value in Cents
Dimes	50	500
Nickels	70	350
Total		850

Yvonne has 50 dimes and 70 nickels in her piggy bank.

Method 2: Writing an Equation

Let the number of dimes be n.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value in Cents
Dimes	n	$10n$
Nickels	$n+20$	$5(n+20)$
Total		850

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

The total value of the nickels and dimes is \$8.50 (or 850 cents).

Write an equation.

$$5(n+20)+10n=850$$

Solve the equation.

$$\begin{aligned}
5(n+20)+10n &= 850\\
5n+100+10n &= 850\\
5n+10n+100 &= 850\\
15n+\cancel{100} &= 850\\
\cancel{-100} &\quad -100\\
\hline
15n &= 750\\
\frac{1}{\cancel{15}_1}\times \cancel{15}^1 n &= \frac{1}{\cancel{15}_1}\times \cancel{750}^{50}\\
n &= 50
\end{aligned}$$

Add -100 to each side.

Multiply each side by $\frac{1}{15}$.

Yvonne has 50 dimes in her piggy bank.

$$\begin{aligned}
n+20 &= 50+20\\
&= 70
\end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

Example 2

Mrs. Mahr is nine times the age of her daughter. In three years Mrs. Mahr will be five times as old as her daughter. What are their ages now?

Solution

To help you understand the problem, you may wish to simplify it. Here is a similar but simpler problem.

Simpler Problem

Mrs. Mahr is 33 years old and her daughter is 3 years old. How many times her daughter's age is Mrs. Mahr now? In three years how many times her daughter's age will Mrs. Mahr be?

Solution to the Simpler Problem

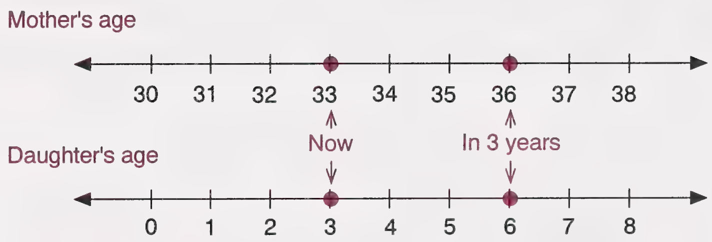
$$33=11\times 3$$

Mrs. Mahr is eleven times her daughter's age now.

$$(33+3)=6\times(3+3)$$

In three years Mrs. Mahr will be six times her daughter's age.

A number line shows the relationship between their ages now and in three years.



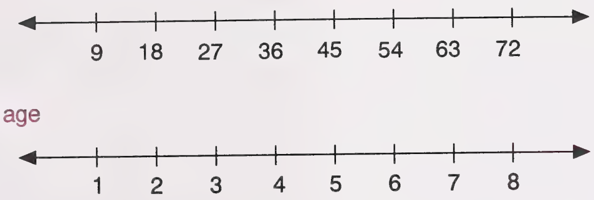
Once you understand the problem, you can solve the problem in several ways.

Method 1: Guess-Check-Revise

Use logic to help make your guess. The mother is 9 times the age of her daughter. The ages can be shown on a number line.

Mother's age

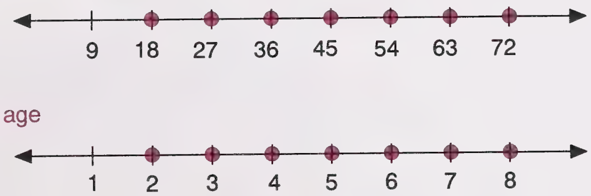
Daughter's age



The mother's present age is probably greater than 18. So the daughter's age is probably greater than 2.

Mother's age

Daughter's age



The problem states that in three years Mrs. Mahr will be five times as old as her daughter.

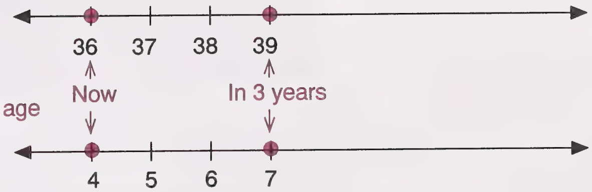
What values will make this statement true?

Guess 1

Mrs. Mahr is 36 and her daughter is 4. So, in three years Mrs. Mahr will be 39 and her daughter will be 7.

Mother's age

Daughter's age



This guess is incorrect because $5 \times 7 \neq 39$.

Guess 2

Mrs. Mahr is 27 and her daughter is 3. So, in three years Mrs. Mahr will be 30 and her daughter will be 6.

Mother's age

Daughter's age

This guess is correct because $5 \times 6 = 30$. This guess satisfies both conditions. Now Mrs. Mahr is nine times her daughter's age, and in three years she will be five times her daughter's age.

So, Mrs. Mahr is 27 and her daughter is 3.

Method 2: Writing an Equation

Let the daughter's age now be n.
Let the mother's age now be $9n$.
In three years the daughter's age will be $n+3$.
In three years the mother's age will be $9n+3$.

You may wish to organize the data in a table.

	Present Age	Age in Three Years
Mrs. Mahr	$9n$	$9n+3$
Daughter	n	$n+3$

In three years Mrs. Mahr will be 5 times as old as her daughter.

Write an equation.

$$9n+3=5(n+3)$$

Solve the equation.

$$\begin{aligned}
9n+3 &= 5(n+3)\\
9n\cancel{+3} &= 5n+15\\
\cancel{-3} & \quad\; -3\\
\hline
9n &= \cancel{5n}+12\\
-5n & \quad \cancel{-5n}\\
\hline
4n &= 12\\
\frac{1}{\cancel{4}_1}\times\cancel{4}^1 n &= \frac{1}{\cancel{4}_1}\times\cancel{12}^3\\
n &= 3
\end{aligned}$$

The daughter is now 3 years old.

$$\begin{aligned}9n &= 9\times 3\\ &= 27\end{aligned}$$

Mrs. Mahr is now 27 years old.

Practice Activity

1. Solve the following problems. You may use any method.

   a. Lori has twice as many dimes as she has quarters. If she has \$9 in dimes and quarters, how many dimes does she have?

   b. Mrs. Wong is presently 9 years older than Mrs. Chan. In eight years, 7 times Mrs. Chan's age will equal 6 times Mrs. Wong's age. What are their ages now?

   c. A pair of track shoes cost twice as much as a pair of running shoes. If two pairs of track shoes and two pairs of running shoes cost \$330, find the cost of each type.

2. Do the puzzle "How Did the Doe Win the Big Animal Race?"[1] that follows.

3. Do the puzzle "What Did They Call the Bug that the Astronauts Brought Back from the Moon?"[2] that follows.

Turn to the Appendix to check your answers.

[1, 2] 1989 Creative Publications for excerpts from *Algebra with Pizzazz.*

HOW DID THE DOE WIN THE BIG ANIMAL RACE?

Solve each problem below. Cross out the box that contains the answer. When you finish, write the letters from the remaining boxes in the squares at the bottom of the page.

1. Mr. Merrill has 3 times as many nickels as dimes. The coins have a total value of $1.50. How many of each coin does he have? _____ nickels, _____ dimes

2. Ms. Lynch has 21 coins in nickels and dimes. Their total value is $1.65. How many of each coin does she have? _____ nickels, _____ dimes

3. A vending machine that takes only dimes and quarters contains 30 coins, with a total value of $4.20. How many of each coin are there? _____ dimes, _____ quarters

4. The total value of the $1 bills and $5 bills in a cash box is $124. There are 8 more $5 bills than $1 bills. How many of each are there? _____ $1 bills, _____ $5 bills

5. A collection of nickels and quarters amounts to $2.60. There are 16 coins in all. How many of each coin are there? _____ nickels, _____ quarters

6. Joe Lick bought some 20-cent and 25-cent stamps. He bought 32 stamps in all, and paid $7.40 for them. How many stamps of each kind did he buy? _____ 20-cent, _____ 25-cent

7. For a school play, 340 tickets valued at $810 were sold. Some cost $2 and some cost $3. How many tickets of each kind were sold? _____ $2 tickets, _____ $3 tickets

8. Romeo bought a mixture of 20-cent, 35-cent, and 50-cent valentines. The number of 20-cent valentines was 1 more than twice the number of 35-cent valentines, and the number of 50-cent valentines was 2 less than the number of 35-cent ones. If he spent $4.20 all together, how many valentines of each kind did he buy? _____ 20-cent, _____ 35-cent, _____ 50-cent

IT 22, 8	SH 6, 10	ER 7, 9	EW 9, 4, 2	EP 13, 6, 4	ON 18, 6	AN 14, 22	AS 240, 100
SE 19, 11	DT 13, 21	EN 210, 130	HE 8, 13	BU 200, 140	KI 12, 20	CK 11, 21	LL 9, 12

WHAT DID THEY CALL THE BUG THAT THE ASTRONAUTS BROUGHT BACK FROM THE MOON?

Solve each problem below. Find your solution at the bottom of the page and cross out the letter above it. When you finish, the answer to the title question will remain.

1. Andy is twice as old as Kate. In 6 years, their ages will total 60. How old is each now?

   Kate _____, Andy _____

2. Mrs. Wang is 23 years older than her daughter. In 5 years, their ages will total 63. How old are they now?

   daughter _____, Mrs. Wang _____

3. Matthew is 3 times as old as Jenny. In 7 years, he will be twice as old as she will be then. How old is each now?

   Jenny _____, Matthew _____

4. Juan is 8 years older than his sister. In 3 years, he will be twice as old as she will be then. How old are they now?

   sister _____, Juan _____

5. Melissa is 24 years younger than Joyce. In 2 years, Joyce will be 3 times as old as Melissa will be then. How old are they now?

   Joyce _____, Melissa _____

6. Tom is 4 years older than Jerry. Nine years ago, Tom was 5 times as old as Jerry was then. How old is each now?

   Jerry _____, Tom _____

7. Kathy is 6 years younger than Bill. Twelve years ago, Bill was twice as old as Kathy was then. How old are they now?

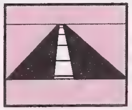
   Bill _____, Kathy _____

8. Dr. Garcia is twice as old as his son. Twenty years ago, he was 4 times as old as his son was then. How old are they now?

   son _____, Dr. Garcia _____

B	A	F	L	E	Y	U	G	N	A	T	O	I	T	C	H	K
5, 13	7, 15	30, 60	14, 28	16, 32	10, 14	33, 66	7, 21	26, 20	9, 32	35, 11	24, 18	8, 24	34, 10	8, 12	15, 38	21, 16

EQUATIONS WITH TWO VARIABLES



What Lies Ahead

In this section you will learn these skills.

- solving equations with two variables using inspection or the guess-check-revise method
- making a table of values
- graphing equations
- writing an equation for a given table of values
- rearranging an equation with two variables

Working Together

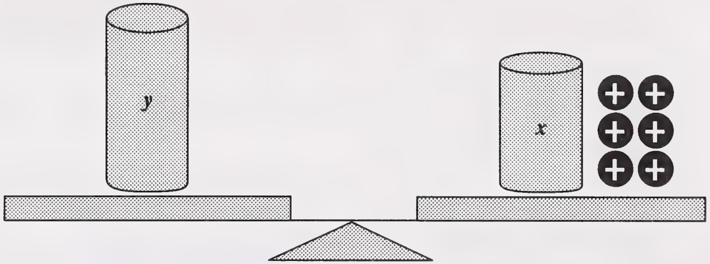
So far in this module you have worked with equations which have only one variable. In this section you will work with equations with two variables.

How do you solve an equation with more than one variable?

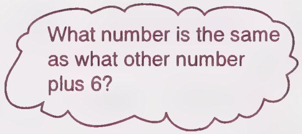
Example: Solve the equation $y = x + 6$.

Solution

Model the equation.

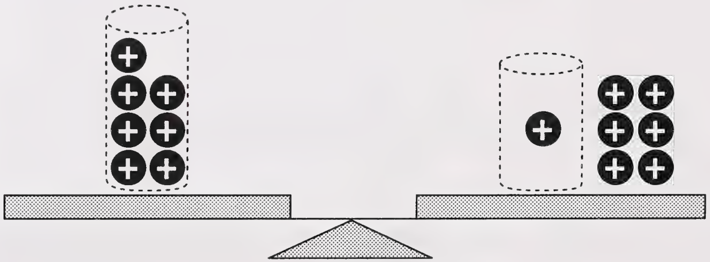


Try to think of values for x and y that will make the equation a true statement.

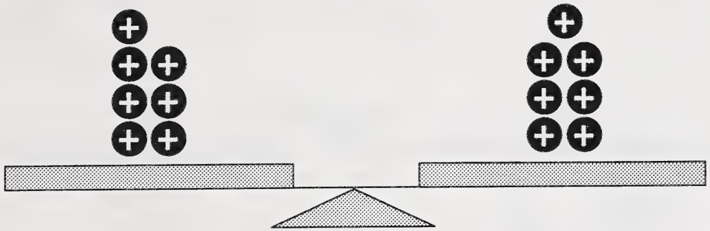


$7 = \boxed{1} + 6$

Verify the solution by replacing y with $+7$ and x with $+1$.



Now simplify both sides.



The result is $+7$ on each side. The scale is balanced.

So, $x = 1$ and $y = 7$ is a solution of $y = x + 6$.

The solution can be written as the ordered pair $(1, 7)$. The first number in the ordered pair shows the x value and the second number in the ordered pair shows the y value.

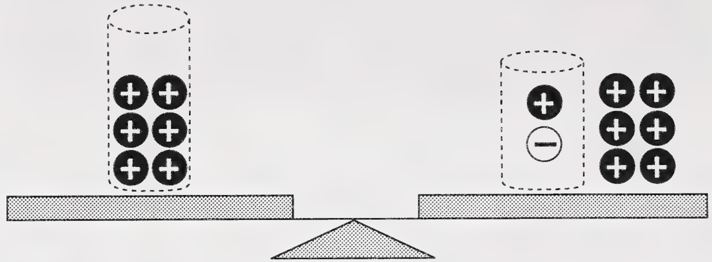
Is $(1, 7)$ the only solution of $y = x + 6$?

Try to think of other values for x and y that will make the equation a true statement.

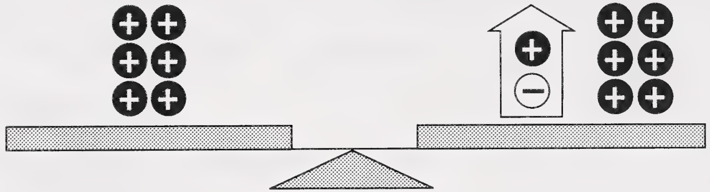
What number is the same as what other number plus 6?

$6 = \boxed{0} + 6$

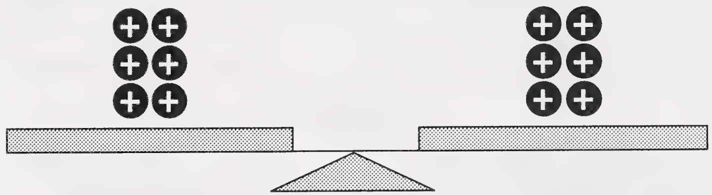
Verify the second solution by replacing x with 0 and y with $+6$.



Now simplify both sides.



The result is $+6$ on both sides.

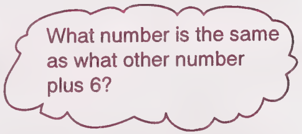


The scale is balanced.

So, $x = 0$ and $y = +6$, or $(0, 6)$, is another solution to the equation $y = x + 6$.

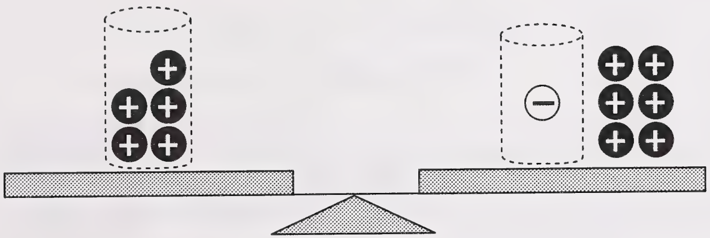
Are there any other solutions?

Try to think of other values for x and y that will make the equation $y = x + 6$ a true statement.

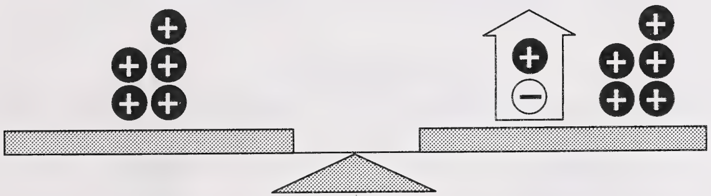


$5 = \boxed{-1} + 6$

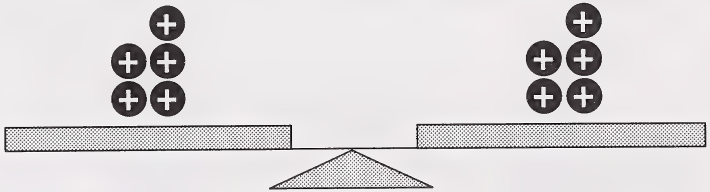
Verify the third solution by replacing x with -1 and y with $+5$.



Then simplify both sides.



The result is $+5$ on each side.



The scale is balanced.

So, $x = -1$ and $y = +5$, or $(-1, 5)$, is another solution.

Practice Activity 1

For each of the equations do the following.

- Use inspection or guess-check-revise methods to find three solutions.
- Write the solutions as ordered pairs.
- Verify the solutions with models.

1. $y = 3x$ **2.** $y = x - 1$ **3.** $y = 2x + 1$

Turn to the Appendix to check your answers.

Working Together

In Practice Activity 1 you used inspection or guess-check-revise methods to find some of the solutions of equations with two variables. You verified the solutions using models.

You can also calculate the solutions of equations with two variables using paper-and-pencil methods.

Example: Solve $y = 2x - 1$.

Solution

Choose a value of x and calculate the corresponding y value.

If $x = -3$, what is y?

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times (-3) - 1 \\ &= -6 - 1 \\ &= -7 \end{aligned}$$

So, one solution is $(-3, -7)$.

If $x = 0$, what is y?

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times 0 - 1 \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

So, another solution is $(0, -1)$.

If $x = 5$, what is y?

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times 5 - 1 \\ &= 10 - 1 \\ &= 9 \end{aligned}$$

So, another solution is $(5, 9)$.

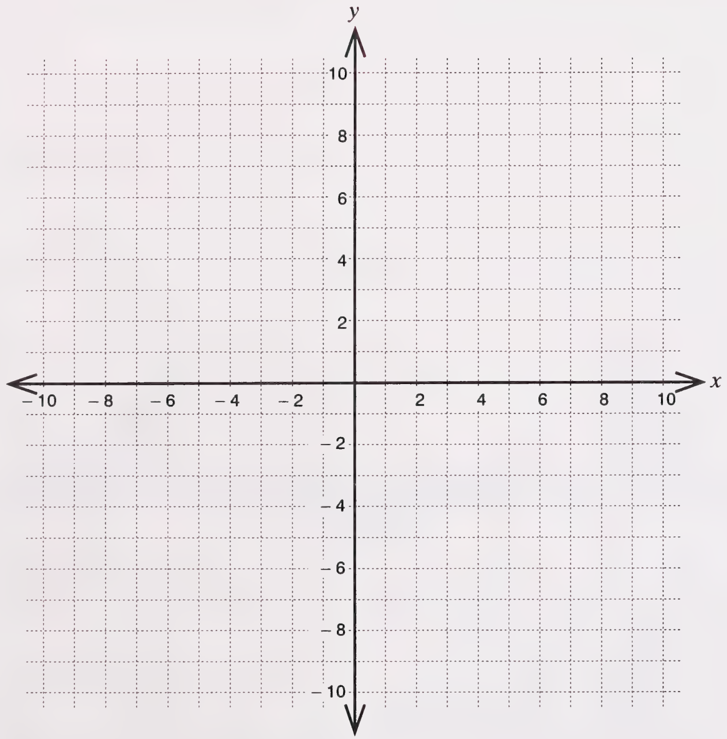
You can continue using this method to find the solutions. However, it is impossible to list all the solutions as there is an infinite number of them.

You can show the solution on a graph.

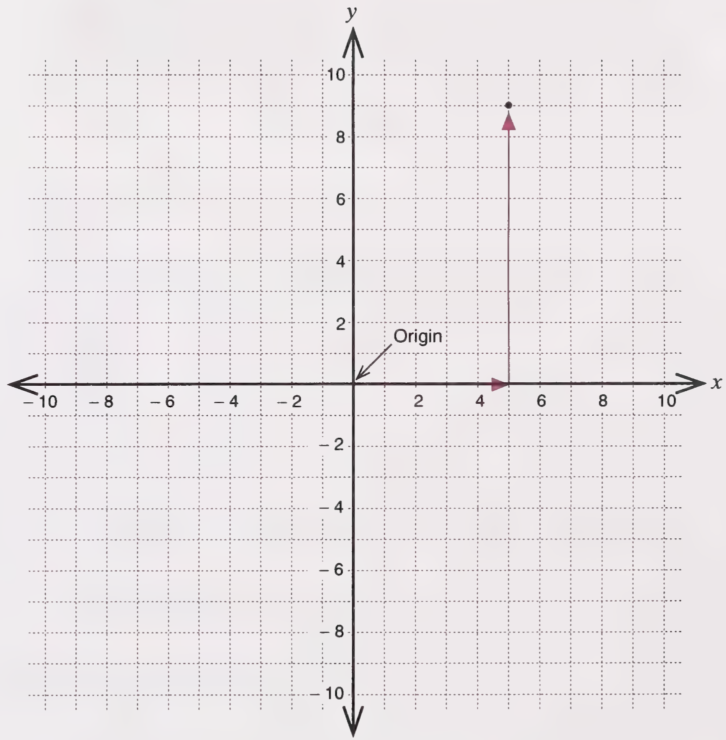
First make a table of values.

x	5	4	3	2	1	0	−1	−2	−3
y	9	7	5	3	1	−1	−3	−5	−7

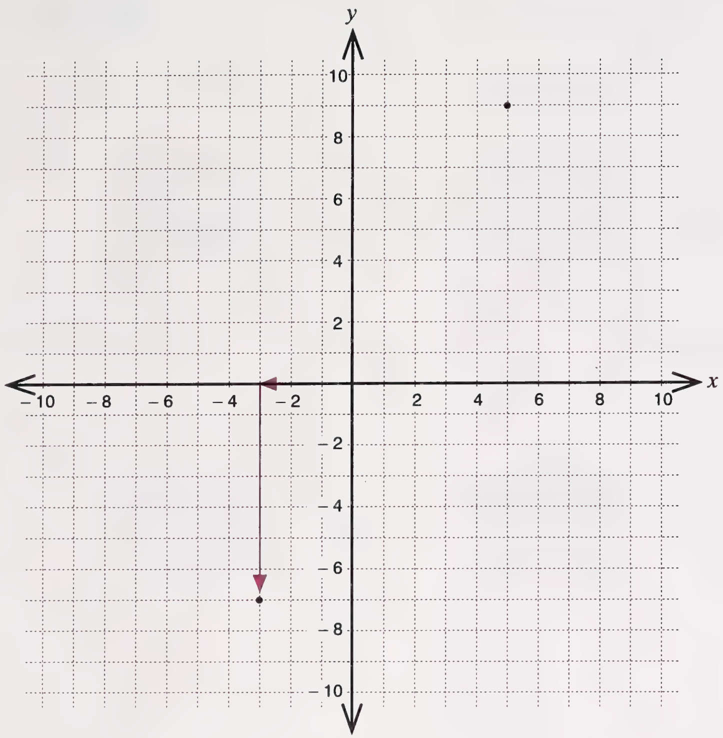
Then label the **horizontal axis** and **vertical axis**



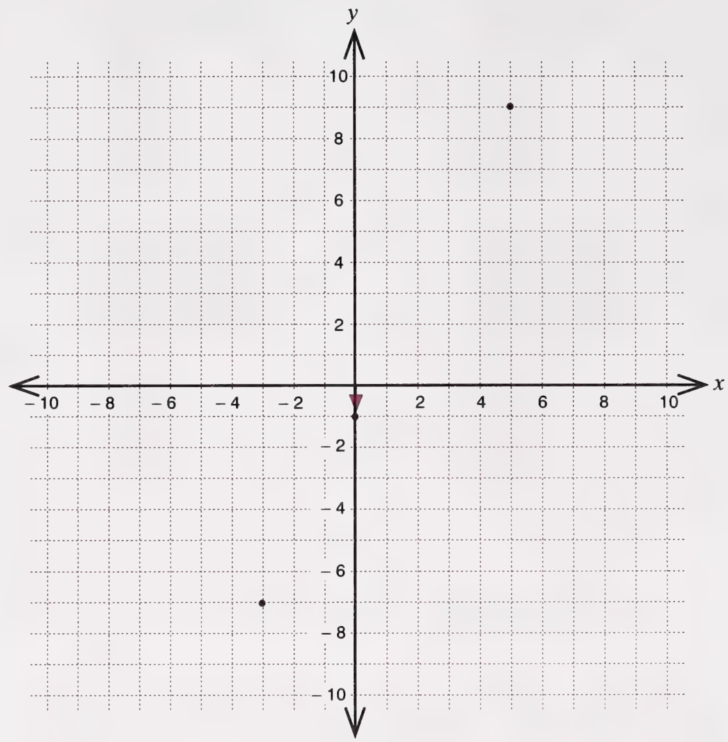
Next plot the ordered pair $(5, 9)$. To do this, begin at the origin and count right 5 and up 9.



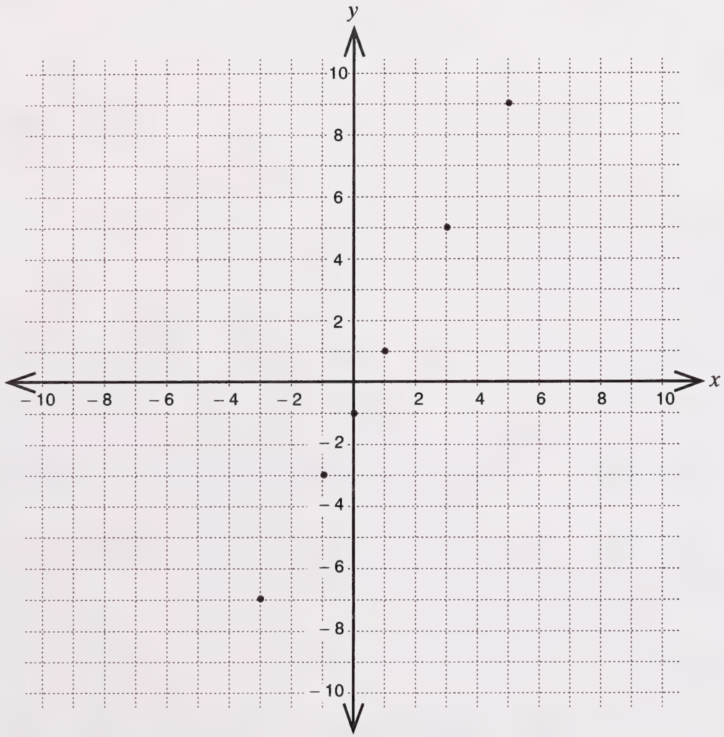
Then plot another ordered pair, such as $(-3, -7)$. To do this, begin at the origin and count left 3 and down 7.



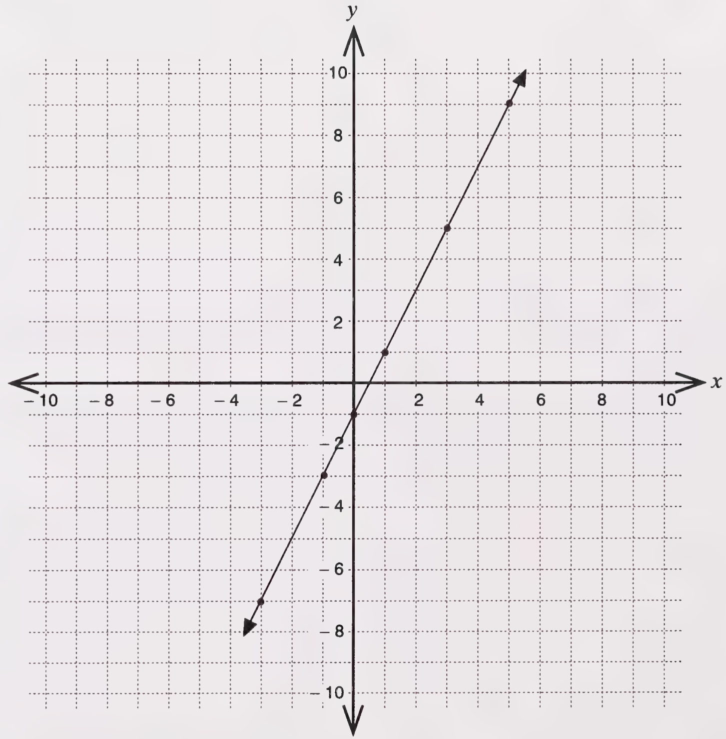
Next plot another ordered pair such as $(0, -1)$. To do this, begin at the origin and count down 1.



Plot a few more ordered pairs, such as $(3, 5)$, $(1, 1)$, and $(-1, -3)$.



Notice that all the coordinates of ordered pairs are in a straight line. Other solutions will fall in this same line. So, connect the coordinates. Put an arrowhead on each end of the line to show that it continues infinitely.



Practice Activity 2

1. Draw and complete the table for each equation on your own paper.

a. $y = -2x$

x	y
1	
4	
−5	
3	

b. $y = 2x + 4$

x	y
3	
−7	
1	
−3	

c. $y = -3x + 1$

x	y
3	
−3	
4	
−2	

d. $y = \frac{1}{2}x - 4$

x	y
10	
−2	
4	
−8	

e. $y = -x + 6$

x	y
4	
−1	
6	
0	

f. $y = -\frac{3}{2}x - 2$

x	y
4	
2	
0	
−2	

g. $y = -3x + 7$

x	y
6	
1	
0	
−2	

h. $y = -x + 1$

x	y
−2	
−9	
9	
6	

2. Graph the equations in Question 1.

3. What do you notice about the graphs of all the equations in Question 1?

Turn to the Appendix to check your answers.

Working Together

Patterns are helpful when completing tables of values.

Example 1: Make a table of values for $y = x + 5$.

Solution

Use consecutive values of x. For example, use 1, 2, 3, 4, 5, and 6.

$x = 1$	$x = 2$	$x = 3$
$y = x + 5$ $= 1 + 5$ $= 6$	$y = x + 5$ $= 2 + 5$ $= 7$	$y = x + 5$ $= 3 + 5$ $= 8$

Then organize the values in a table to discover a pattern.

$y = x + 5$		
x	y	Pattern
1	6	
2	7	+ 1
3	8	+ 1

Apply the pattern to find the last three values.

$y = x + 5$		
x	y	Pattern
1	6	
2	7	+ 1
3	8	+ 1
4	9	+ 1
5	10	+ 1
6	11	+ 1

Practice Activity 3

1. Draw and complete the following tables of values on your own paper. Evaluate the first three values and then use a pattern to help you to find the last three values.

a.

$y = x + 2$	
x	y
0	
1	
2	
3	
4	
5	

b.

$y = 4x - 1$	
x	y
0	
1	
2	
3	
4	
5	

c.

$y = 2x$	
x	y
0	
1	
2	
3	
4	
5	

d.

$y = 3x + 2$	
x	y
0	
1	
2	
3	
4	
5	

2. How are the patterns in Question 1 similar to the equations?

3. Use each of the following tables of values to find each value of $\square$.

a.

$y = \square\, x - 3$	
x	y
0	−3
1	1
2	5
3	9
4	13

b.

$y = \square\, x + 1$	
x	y
0	1
1	7
2	13
3	19
4	25

c.

$y = \square\, x - 1$	
x	y
0	−1
1	1
2	3
3	5
4	7

d.

$y = \square\, x + 4$	
x	y
0	4
1	7
2	10
3	13
4	16

e.

$y = \square\, x + \square$	
x	y
0	3
1	5
2	7
3	9
4	11

f.

$y = \square\, x - \square$	
x	y
0	−4
1	−1
2	2
3	5
4	8

g.

$y = \square\, x + \square$	
x	y
0	1
1	3
2	5
3	7
4	9

h.

$y = \square$	
x	y
0	−3
1	2
2	7
3	12
4	17

i.

$y =$	
x	y
0	1
1	4
2	7
3	10
4	13

Turn to the Appendix to check your answers.

Working Together

Sometimes equations with two variables are written in the form of $y = mx + b$.

Examples

$y = 3x + 4$ $\qquad$ $y = -2x - 5$

At other times you may be given the equation in another form and you may need to solve for y.

Example 1: Solve $x + y = 2$ for y.

Solution

To solve for y, add $-x$ to each side of the equation.

$$\begin{array}{rcl} \cancel{x} + y & = & 2 \\ \cancel{-x} & & -x \\ \hline y & = & -x + 2 \end{array}$$

So, $y = -x + 2$.

Example 2: Solve $y - 3x = 4$ for y.

Solution

To solve for y, add $+3x$ to each side of the equation.

$$\begin{array}{rcl} y \cancel{- 3x} & = & 4 \\ \cancel{+3x} & & 3x \\ \hline y & = & 3x + 4 \end{array}$$

So, $y = 3x + 4$.

Example 3: Solve $2y = 3x + 4$ for y.

Solution

To solve for y, multiply each side of the equation by $\frac{1}{2}$.

$$\frac{1}{\cancel{2}_1} \times \cancel{2}^1 y = \frac{1}{2}(3x + 4)$$

$$y = \frac{3}{2}x + 2$$

So, $y = \frac{3}{2}x + 2$.

Example 4: Solve $3y - 5 = 2x + 1$ for y.

Solution

To solve for y, add 5 to each side of the equation.

$$\begin{array}{rl} 3y \cancel{-5} & = 2x + 1 \\ \cancel{+5} & \quad\ \ + 5 \\ \hline 3y \quad & = 2x + 6 \end{array}$$

Next multiply each side by $\frac{1}{3}$.

$$\frac{1}{\cancel{3}_1} \times \cancel{3}^1 y = \frac{1}{3}(2x + 6)$$

$$y = \frac{2}{3}x + 2$$

So, $y = \frac{2}{3}x + 2$.

Practice Activity 4

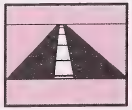
1. Solve the following equations for y.

   a. $y - x = 2$
   b. $y + 3 = x - 4$
   c. $2y = 5x - 8$
   d. $3y + 1 = 6x + 7$

2. Graph the equations in Question 1.

Turn to the Appendix to check your answers.

RELATIONS



What Lies Ahead

In this section you will learn these skills.

- describing a relation using a table, a rule, ordered pairs, and a graph
- using formulas to solve problems
- rearranging formulas

Working Together

Algebra has many practical applications.

In this section you will learn how to describe the relationship between pairs of numbers in several ways.

Example 1

Mark and Carl are brothers. When Carl was 1, Mark was 3. When Carl was 2, Mark was 4. How are their ages related?

The relationship can be expressed in several ways.

- The relationship can be shown in a table.

Carl's Age (c)	Mark's Age (m)	Relation
1	3	$1+2$
2	4	$2+2$
3	5	$3+2$
4	6	$4+2$
5	7	$5+2$

- The relationship can be generalized in words.

  Mark is two years older than Carl.

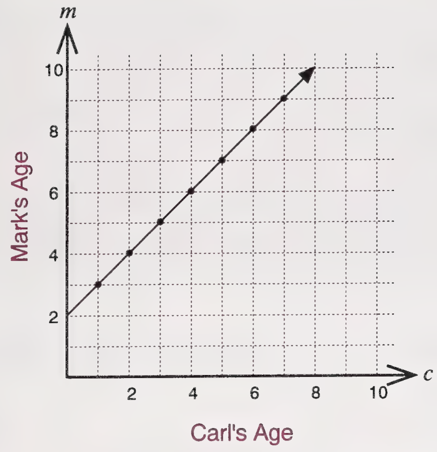
- The relationship can also be described using an equation.

$$m = c + 2$$

- The relationship between Carl's age and Mark's age can be shown by **ordered pairs**.

$$(1,3),\ (2,4),\ (5,7),\ (6,8),\ (7,9), \ldots$$

- The relationship can be described by a **graph**.



Example 2

A piggy bank contains only dimes. How is the total amount of money in the bank related to the number of dimes?

The relationship can be described in several ways.

- The relationship can be shown in a table.

Number of Dimes (n)	Value in Cents (v)	Relation
1	10	$10 = 10 \times 1$
2	20	$20 = 10 \times 2$
3	30	$30 = 10 \times 3$
4	40	$40 = 10 \times 4$
5	50	$50 = 10 \times 5$

- The relationship can be generalized in words.

  The value in cents is ten times the number of dimes.

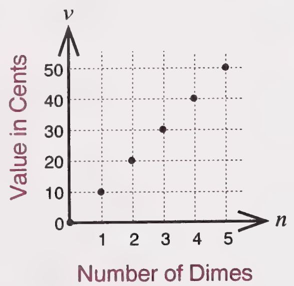
- The relationship can also be described using an equation.

$$v = 10n$$

- The relationship between the number of dimes and value in cents can be described by ordered pairs.

$$(0,1),\ (1,10),\ (2,20),\ (3,30),\ (4,40),\ (5,50), \ldots$$

- The relationship can be described by a graph.



Note: The points on this graph are not joined by a line because fractional values are not possible.

Practice Activity 1

1. Reema likes to go horseback riding. How is the cost related to the riding time?

Riding Time in Hours (t)	Cost in Dollars (c)	Relation
1	6	$6 = 4 + 2 \times 1$
2	8	$8 = 4 + 2 \times 2$
3	10	$10 = 4 + 2 \times 3$
4	12	$12 = 4 + 2 \times 4$
5	14	$14 = 4 + 2 \times 5$

Describe the relationship by using each of the following methods.

a. Write the words to descibe the relation.

b. Write an equation to describe the relation

c. Write ordered pairs to describe the relation.

d. Describe the relationship using a graph.

2. How is Vlad's hourly pay related to Maria's hourly pay?

Maria's Pay (m)	Vlad's Pay (v)	Relation
5	4	$4 = 5 - 1$
6	5	$5 = 6 - 1$
7	6	$6 = 7 - 1$
8	7	$7 = 8 - 1$
9	8	$8 = 9 - 1$

Describe the relationship by using each of the following methods.

a. Write the words to descibe the relation.

b. Write an equation to describe the relation

c. Write ordered pairs to describe the relation.

d. Use a graph to describe the relation.

Turn to the Appendix to check your answers.

Working Together

Some equations are used over and over again. These equations are often called **formulas**.

Example 1

Scientists can estimate the height of an individual from one complete section of bone.

The formula shown below is used to find the height of a female (H) when the length of the humerus (h) is known.

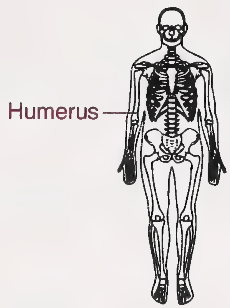
$$H = 2.75h + 71.4$$

If the length of the humerus of a female is 30 cm, how tall is she?

Solution

If $h = 30$, then

$$\begin{aligned} H &= 2.75h + 71.4 \\ &= 2.75 \times 30 + 71.4 \\ &= 82.5 + 71.4 \\ &= 153.9 \end{aligned}$$



The female is about 154 cm tall.

Example 2

When a new drug is developed and the correct dosage for adults is determined, it is important to find the corresponding dosage for children.

The formula shown below is used to find the correct child's dosage (c) when the child's mass (m) and the adult dosage (a) is known.

$$c = \frac{m}{68} \times a$$

If a child's mass is 16 kg and adult dosage is 15 mL, what is the correct child's dosage?

Solution

If $m = 16$ and $a = 15$, then

$$\begin{aligned} c &= \frac{m}{68} \times a \\ &= \frac{16}{68} \times 15 \\ &= 3.5 \end{aligned}$$

The correct child's dosage is 3.5 mL.

Example 3

The mass of a freshly caught fish can be found by the following formula. It relates the mass (m) in kilograms to the length (l) in centimetres and the girth (g) in centimetres.

$$m = \frac{\ell \times g^2}{24\,600}$$

Girth is the distance around the thickest part of the fish.

What is the mass of a salmon that is 82 cm long with a girth of 50 cm?

Solution

If $\ell = 82$ and $g = 50$, then

$$\begin{aligned} m &= \frac{\ell \times g^2}{24\,600} \\ &= \frac{82 \times 50^2}{24\,600} \\ &= \frac{82 \times 2500}{24\,600} \\ &= 8.3 \end{aligned}$$

The mass of the salmon is 8.3 kg.

Rearranging Formulas

It is sometimes necessary to rearrange a formula to find the value of a variable.

Example 1

The formula shown below is used to find the distance travelled (d) when the average rate of speed (v) and the time travelled (t) is known.

$$d = vt$$

If a driver travelled 400 km in 5 h, what was the average rate of speed?

Solution

Method 1

Solve for v and then substitute the known values.

$$\begin{aligned} d &= vt \\ \frac{d}{t} &= \frac{v\cancel{t}^{1}}{\cancel{t}_{1}} \\ \frac{d}{t} &= v \end{aligned}$$

Divide each side by t to isolate v.

So, $v = \frac{d}{t}$.

If $d = 400$ and $t = 5$, then

$$\begin{aligned} v &= \frac{d}{t} \\ &= \frac{400}{5} \\ &= 80 \end{aligned}$$

The driver travelled at an average rate of speed of 80 km/h.

Method 2

Substitute the known values and solve for v.

If $d = 400$ and $t = 5$, then

$$d = vt$$
$$400 = v \times 5$$

Solve for v.

$$\frac{400}{5} = \frac{v\cancel{t}^{1}}{\cancel{t}_{1}}$$

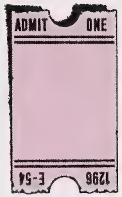
Divide each side by t to isolate v.

$$80 = v$$

The driver travelled at an average rate of speed of 80 km/h.

Example 2

The formula shown below is used to find the cost (c) in dollars of printing a number (n) of hockey tickets.



$$c = 50 + 0.1n$$

If the printing cost is \$350, how many tickets were printed?

Solution

Method 1

Solve for n and then substitute the known value.

$$\begin{aligned} c \quad &= \cancel{50} + 0.1n \\ -50 \quad & \quad \cancel{-50} \\ \hline c - 50 &= \quad 0.1n \end{aligned}$$

Add -50 to each side.

$$\frac{c-50}{0.1} = \frac{\cancel{0.1}^{1}n}{\cancel{0.1}_{1}}$$

Divide each side by 0.1.

$$\frac{c-50}{0.1} = n$$

So, $n = \frac{c-50}{0.1}$.

If $c = 350$, then

$$\begin{aligned} n &= \frac{c - 50}{0.1} \\ &= \frac{350 - 50}{0.1} \\ &= \frac{300}{0.1} \\ &= 3000 \end{aligned}$$

There were 3000 tickets printed.

Method 2

Substitute the known value and solve for n.

If $c = 350$,

$$\begin{aligned} c &= 50 + 0.1n \\ 350 &= 50 + 0.1n \end{aligned}$$

Solve for n.

$$\begin{aligned} 350 &= \cancel{50} + 0.1n \\ -50 &\quad \cancel{-50} \\ \hline 300 &= 0.1n \\ \frac{300}{0.1} &= \frac{0.1n}{0.1} \\ 3000 &= n \end{aligned}$$

Add -50 to each side.

Divide each side by 0.1.

There were 3000 tickets printed.

Example 3

The formula below gives the apprriximate distance (d) in metres that an object will fall in t seconds.

$$d = 5t^2$$

How long will an object take to fall 180 m?

Solution

Method 1

Solve for t and then substitute the known value.

$$\begin{aligned} d &= 5t^2 \\ \frac{1}{5} \times d &= \frac{1}{\cancel{5}_1} \times \cancel{5}^1 t^2 \\ \frac{d}{5} &= t^2 \\ \sqrt{\frac{d}{5}} &= \sqrt{t^2} \\ \sqrt{\frac{d}{5}} &= t \end{aligned}$$

Multiply each side by $\frac{1}{5}$.

Take the square root of each side.

So, $t = \sqrt{\frac{d}{5}}$.

If $d = 180$, then

$$\begin{aligned} t &= \sqrt{\frac{180}{5}} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

It will take 6 s for an object to fall 180 m.

Method 2

Substitute the known value and solve for t.

If $d = 180$, then

$$d = 5t^2$$
$$180 = 5t^2$$
$$\frac{180}{5} = \frac{\cancel{5}^{1} t^2}{\cancel{5}_{1}}$$
← Divide each side by 5.
$$36 = t^2$$
$$\sqrt{36} = \sqrt{t^2}$$
← Take the square root of each side.
$$6 = t$$

It will take 6 s for an object to fall 180 m.

Practice Activity 2

1. When money is borrowed, there is usually a charge. This charge is called the interest (I) and the amount borrowed is called the principal (P). The amount of money paid to the lender (A) is given by this formula.

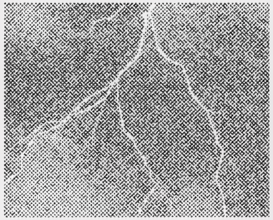
$$A = P + I$$

   a. If the principal is $1000 and the interest is $90, what is the amount paid to the lender?

   b. If the amount paid to the lender is $1620 and the interest is $120, what is the principal?

2. Sound travels at about 343 m/s is at 20°C. The formula below can be used to find the distance (d) in metres that sound travels in t seconds at 20°C.

$$d = 343t$$



   a. If there is a 6-s interval between the time Fatima sees lightning and hears thunder, find the distance Fatima is from the storm.

   b. If Randall is 5 km from a storm, what will be the time interval between the time he sees lightning and hears thunder? **Hint:** 1 km = 1000 m

3. Did you know that you can find the temperature by counting the chirps of a cricket? The formula below relates the temperature (t) in degrees Celsius for the number of chirps (n) in one minute.

$$t = \frac{n}{8} + 5$$

   a. Find the temperature if Ruth counts 80 cricket chirps in one minute.

   b. How many times should a cricket chirp at a temperature of 25°C?

4. The approximate braking distance for a car on dry pavement is given by the following formula. It relates the braking distance (d) in metres and the rate of speed (v) in kilometres per hour.

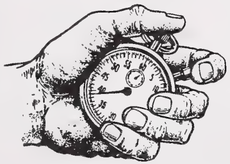
$$d = \frac{v^2}{210}$$

a. Find the approximate braking distance at 80 km/h.

b. There was a car accident. The police officer who arrived at the scene measured the skid marks and found that the braking distance was 62 m. What was the speed of the driver?

5. The time (t) in seconds required for the swing of a pendulum is given by the following formula. It relates the time and the length (ℓ) of the pendulum in centimetres.

$$t = 0.2\sqrt{\ell}$$



a. Find the time required for the swing of pendulum that is 64 cm long.

b. Find the length of a pendulum that makes one swing in 2 s.

Turn to the Appendix to check your answers.

Did You Know?

Emmy Noether

Emmy Noether was a great woman of the twentieth century. She had many troubles during her life.

She was born of Jewish parents in Germany in 1882. As a child she was taught to sew, cook, and dance. These were the things everyone thought girls ought to learn. She learned some other things too. Her father was a professor of mathematics, and he often invited his friends to dinner. Young Emmy would listen to them talk about problems, and soon she was able to join in their talk.

A professor from the university came to the Noether home and taught Emmy about mathematics. She was very bright and was soon able to do many problems herself.

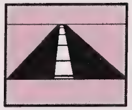
Later she wanted to teach at the university. No woman had ever been allowed to have that kind of job. Even though she was a very good mathematician and teacher, it was a long time before she was able to get the teaching job. When she finally did, she was a popular teacher who shared her ideas with everyone.

When the Nazis took over Germany, she had more troubles. Because Emmy Noether was a Jew who spoke out against war, the Nazis ran her out of the country. She went to the United States, where her work was appreciated and where she easily became a professor.

In spite of her great mind and her troubles, she was a modest, kind person. Someone who knew her well said she was "warm like a loaf of bread." When Emmy Noether died in 1935, the world lost a truly wonderful and a truly wonderful mathematician.[1]

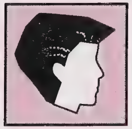
[1] The National Council of Teachers of Mathematics for the excerpt from *Mathematical History*.

MODULE CONCLUSION



What Lies Ahead

In the module conclusion you will review the module and do the module assignment.



Working Together

In Module 4 the following skills were taught.

- interpreting an equation
- interpreting a conditional equation
- modelling equations
- solving equations by inspection or by using the guess-check-revise method
- verifying solutions to equations
- translating English sentences into equations
- interpeting an inequation
- modelling inequations
- solving inequations by inspection or by using the guess-check-revise method
- verifying solutions to inequations
- using formal procedures to solve equations of the form $x + a = b$, $ax = b$, $\frac{x}{a} = b$, $\frac{x}{a} = \frac{b}{c}$, $\frac{a}{x} = \frac{b}{c}$, $ax + bx = c$, $a(x + b) = c$, $ax + b = cx$, $ax + b = cx + d$, $x^2 + a = b$, $ax^2 = b$, $\sqrt{x} + a = b$, and $a\sqrt{x} = b$.
- using formal procedures to solve inequations of the form $x + a < b$, $x + a > b$, $ax < b$, and $ax > b$
- solving equations with two variables using inspection or the guess-check-revise method
- making a table of values
- graphing equations
- writing an equation for a given table of values
- rearranging an equation with two variables
- describing a relation using a table, a rule, ordered pairs, and a graph
- using formulas to solve problems
- rearranging formulas

Turn to Section 1 and review the Pretest. Correct any errors you may have made at the time you did the Pretest.

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment. You may use your notes, but do the assignment independently.

Afterwards, submit the assignment for a grade and feedback from your learning facilitator.

APPENDIX

- Glossary
- Suggested Answers
- Cut-out Learning Aids

Glossary

Additive inverse: the opposite of a number in an expression. The following numbers are additive inverses.

$+3$ and -3 $\qquad$ $5a$ and $-5a$

Algebra: the branch of mathematics which describes basic arithmetic relations

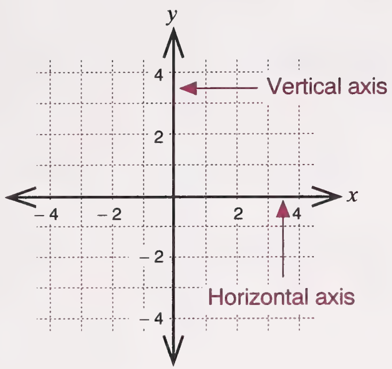
Algebraic equation: an equation with a variable

$2a = 6$ is an algebraic equation.

Algebraic expression: a combination of numerals, variables, and/or other mathematical symbols

$3a + 2$ is an algebraic expression.

Axis (plural axes): either of the intersecting number lines of a graph



Cartesian plane: a number grid on a plane

Conditional equation: an equation with a variable

$a + 5 = 8$ is a conditional equation.

Coordinates: the two numbers in an ordered pair that locates a point

Coordinate plane: a number grid on a plane

Cross products: the products that result from cross multiplying

$\frac{a}{b} \times \frac{c}{d}$

ad and bc are the cross products.

Equation: a number sentence showing that the left-hand side and the right-hand side are equal

$7 + 3 = 2 + 8$ is an equation.

Formula: an equation that is used frequently

Graph: a pictorial device that displays a relationship

Inequation: a number sentence showing that the left-hand side and the right-hand side are not equal

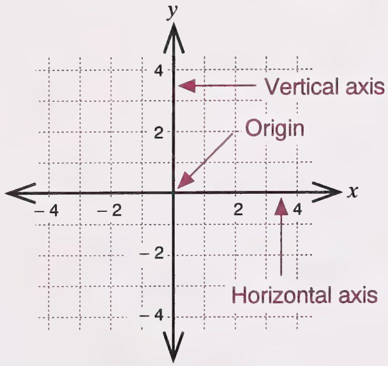
Multiplicative inverse: the reciprocal of a number in an expression. The following numbers are multiplicative inverses.

9 and $\frac{1}{9}$ $\qquad$ a and $\frac{1}{a}$

The product of two multiplicative inverses is 1.

Ordered pair: a pair of numbers in which order is important

Origin: the point where the vertical axis and the horizontal axis of a graph intersect



Plot: to locate a point on a grid by means of coordinates

Solution: a value of the variable which when used in place of the variable in an equation or inequation gives a true statement

$r = 3$ is the solution of $r + 1 = 4$.
$r < 3$ is the solution of $r + 1 < 4$.

Variable: a symbol (usually a letter) used to represent an unspecified or unknown number

Verifying: checking the solution

Suggested Answers

Section 1: Pretest

See your learning facilitator to check your answers.

Section 2: Practice Activity 1

1. a. 8 added to what number is 12?

$\boxed{4} + 8 = 12$

So, $n = 4$.

b. 5 subtracted from what number is -5?

$\boxed{0} - 5 = -5$

So, $p = 0$.

c. 4 times what number is 20?

$4 \times \boxed{5} = 20$

So, $b = 5$.

d. 3 times what number is -12?

$3 \times \boxed{-4} = -12$

So, $b = -4$.

e.

8 plus 3 times what number is 5?

$3 \times \boxed{-1} + 8 = 5$

So, $w = -1$.

f.

1 subtracted from 4 times what number is 7?

$4 \times \boxed{2} - 1 = 7$

So, $t = 2$.

g.

4 added to 5 times what number is 10 added to 4 times what number?

$5 \times \boxed{6} + 4 = 4 \times \boxed{6} + 10$

So, $x = 6$.

h.

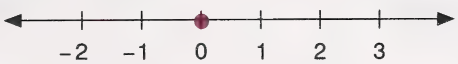
Two times 7 less than what number is 4?

$2(\boxed{9} - 7) = 4$

So, $a = 9$.

2. a.

b.



c.

d.

e.

f.

g.

h.

Section 2: Practice Activity 2

1. **a.** $12 - n = 4$ **b.** $n + \frac{n}{2} = 48$ **c.** $n - 2 = 7$

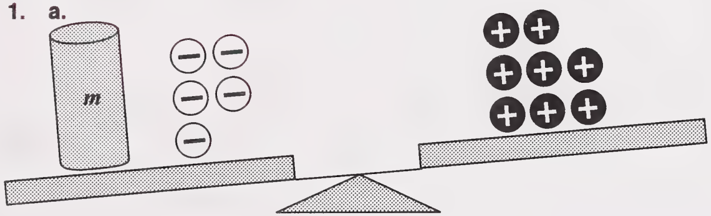
   d. $r + 3 = 52$ **e.** $2m + 5 = 79$ **f.** $\frac{n}{2} - 9 = 27$

2. **a.** $5n + 8 = 38$ **b.** $5(n + 8) = 55$ **c.** $5(n + 8) = 45$

   d. $8n + 5 = 29$ **e.** $8(n + 5) = 30$ **f.** $2(5n + 8) = 24$

Section 3: Practice Activity 1

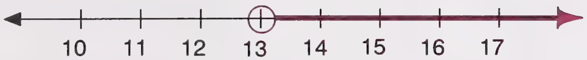
1. a.



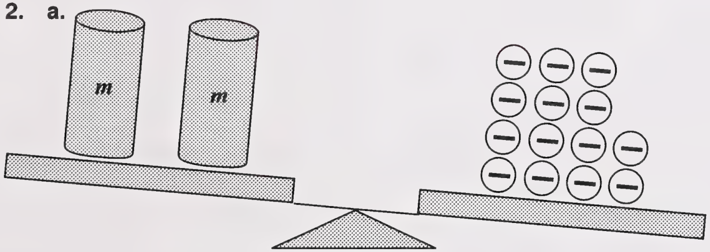
b. Is $13-5>8$? No; LS $\ngtr$ RS.
Is $14-5>8$? Yes; LS > RS.
Is $15-5>8$? Yes; LS > RS.
Is $16-5>8$? Yes; LS > RS.

So, $m>13$.

c.



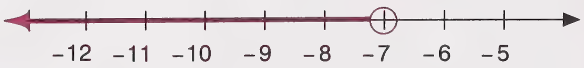
2. a.



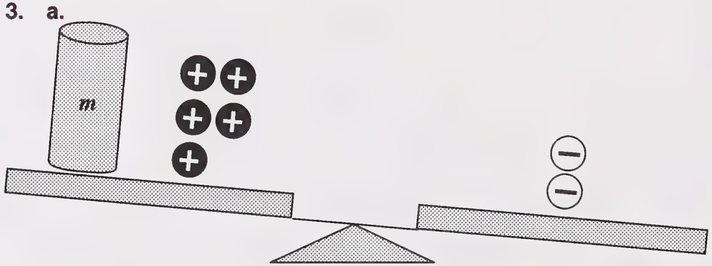
b. Is $2\times(-7)<-14$? No; LS $\nless$ RS.
Is $2\times(-8)<-14$? Yes; LS < RS.
Is $2\times(-9)<-14$? Yes; LS < RS.
Is $2\times(-10)<-14$? Yes; LS < RS.

So, $m<-7$.

c.



3. a.

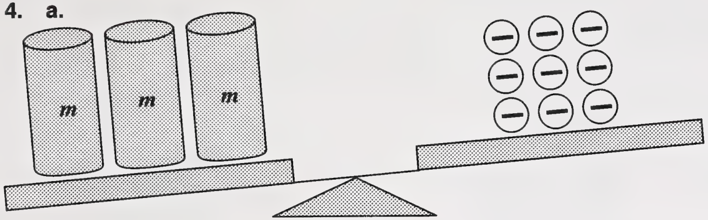


b. Is $(-7)+5<-2$? No; LS $\nless$ RS.
Is $(-8)+5<-2$? Yes; LS < RS.
Is $(-9)+5<-2$? Yes; LS < RS.
Is $(-10)+5<-2$? Yes; LS < RS.

So, $m<-7$.

c.

4. a.



b. Is $3 \times (-3) > -9$? No; LS $\ngtr$ RS.
Is $3 \times (-2) > -9$? Yes; LS > RS.
Is $3 \times (-1) > -9$? Yes; LS > RS.
Is $3 \times 0 > -9$? Yes; LS > RS.

c.

Section 3: Practice Activity 2

1. a. Is $3+3<6$? No; LS $\nless$ RS.
Is $2+3<6$? Yes; LS < RS.
Is $1+3<6$? Yes; LS < RS.

So, $x<3$.

b. Is $7-4>3$? No; LS $\ngtr$ RS.
Is $8-4>3$? Yes; LS > RS.
Is $9-4>3$? Yes; LS > RS.

So, $k>7$.

c. Is $3 \times 2 < 6$? No; LS $\nless$ RS.
Is $3 \times 1 < 6$? Yes; LS < RS.
Is $3 \times 0 < 6$? Yes; LS < RS.

So, $k<2$.

d. Is $5 \times 2 > 10$? No; LS $\ngtr$ RS.
Is $5 \times 3 > 10$? Yes; LS > RS.
Is $5 \times 4 > 10$? Yes; LS > RS.

So, $x>2$.

2. a.

b.

c.

d.

Section 4: Practice Activity 1

1. a. -2 b. -4 c. -9 d. $+5$ e. $+2$

2. a.
$$\begin{array}{rcr} x+\cancel{2} &=& 7 \\ \cancel{-2} && -2 \\ \hline x &=& 5 \end{array}$$

b.
$$\begin{array}{rcr} s+\cancel{4} &=& 9 \\ \cancel{-4} && -4 \\ \hline s &=& 5 \end{array}$$

c.
$$\begin{array}{rcr} m+\cancel{9} &=& -13 \\ \cancel{-9} && -9 \\ \hline m &=& -22 \end{array}$$

d. $$\begin{array}{rcr} t \cancel{-5} & = & 7 \\ \cancel{+5} & & +5 \\ \hline t & = & 12 \end{array}$$

e. $$\begin{array}{rcr} y \cancel{-2} & = & -8 \\ \cancel{+2} & & +2 \\ \hline y & = & -6 \end{array}$$

Section 4: Practice Activity 2

1. a.

Start s → Subtract 3 → End 5

End s ← Add 3 ← Start 5

So, $s = 8$.

b.

Start k → Add 12 → End 39

End k ← Subtract 12 ← Start 39

So, $k = 27$.

c.

Start n → Add 2 → End 8

End n ← Subtract 2 ← Start 8

So, $n = 6$.

d.

Start s → Subtract 4 → End 3

End s ← Add 4 ← Start 3

So, $s = 7$.

2. a. $$\begin{array}{rcr} n \cancel{-8.5} & = & 12.3 \\ \cancel{+8.5} & & +8.5 \\ \hline n & = & 20.8 \end{array}$$

b. $$\begin{array}{rcr} q \cancel{-\frac{1}{2}} & = & \frac{3}{4} \\ \cancel{+\frac{1}{2}} & & +\frac{1}{2} \\ \hline q & = & \frac{5}{4} \end{array}$$

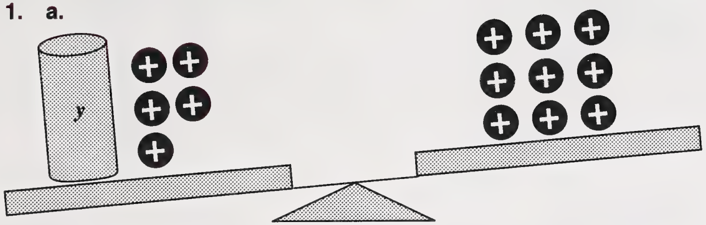
c. $$\begin{array}{rcr} p \cancel{+7.5} & = & 8.2 \\ \cancel{-7.5} & & -7.5 \\ \hline p & = & 0.7 \end{array}$$

d. $$\begin{array}{rcr} m \cancel{-5\frac{3}{4}} & = & 3\frac{1}{4} \\ \cancel{+5\frac{3}{4}} & & +5\frac{3}{4} \\ \hline m & = & 9 \end{array}$$

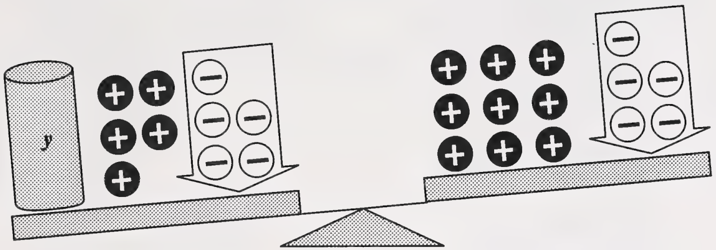
3. QUESTIONABLE

Section 5: Practice Activity 1

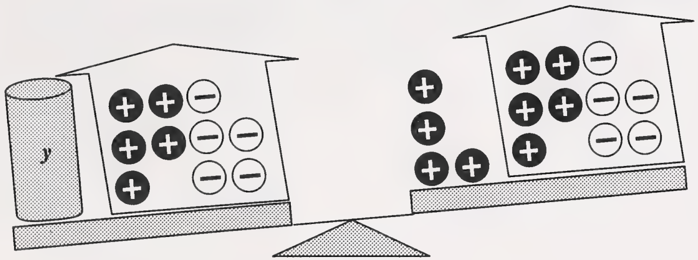
1. a.



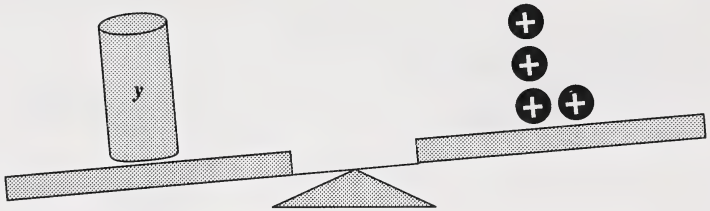
Add -5 to each side.



Simplify by removing the zeros.

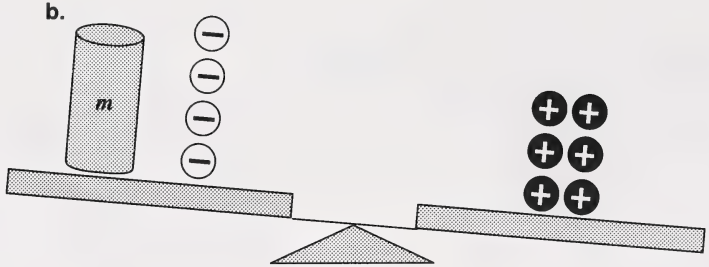


The result is this.

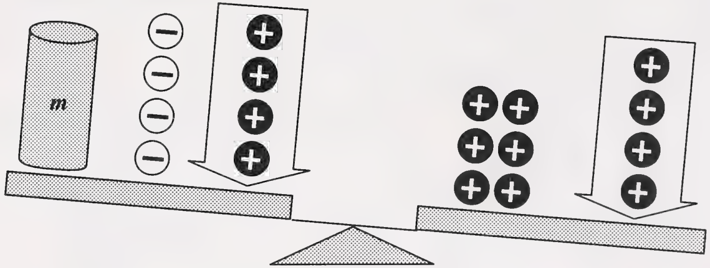


So, $y > 4$.

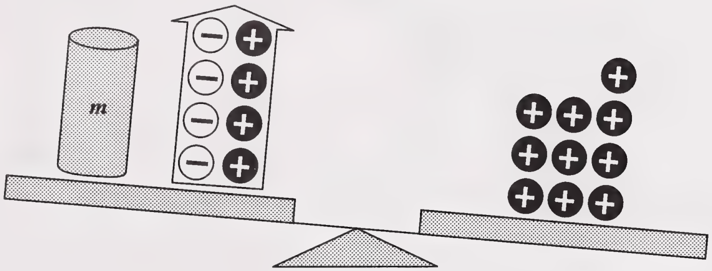
b.



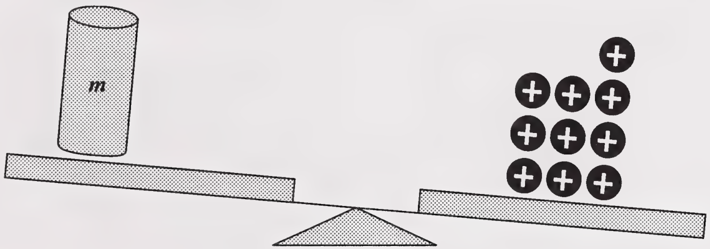
Add 4 to each side.



Simplify by removing the zeros.

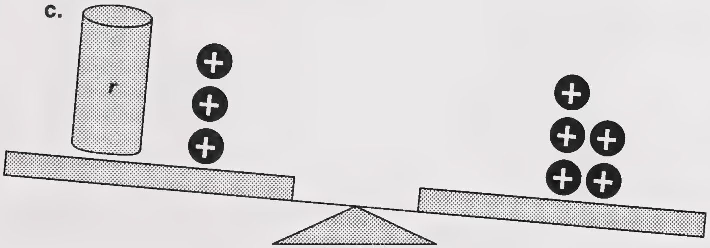


The result is this.

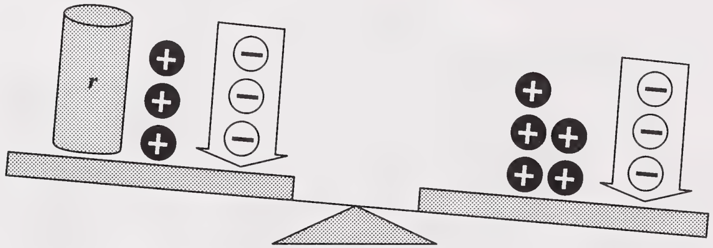


So, $m < 10$.

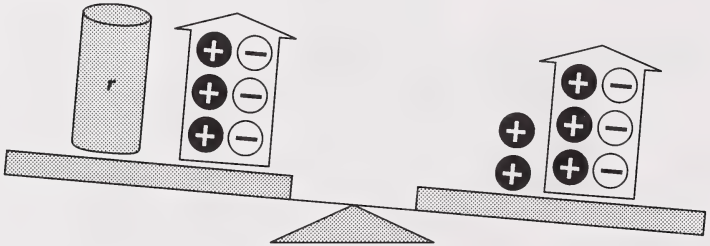
c.



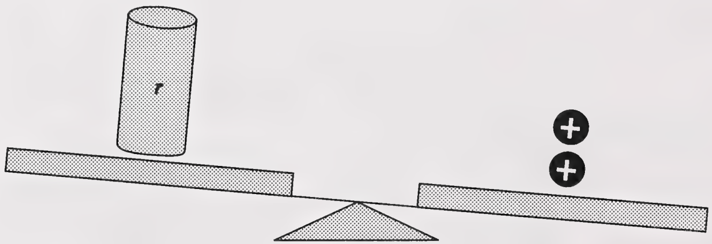
Add -3 to each side.



Simplify by removing the zeros.

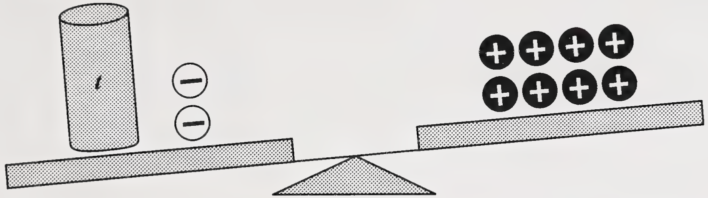


The result is this.

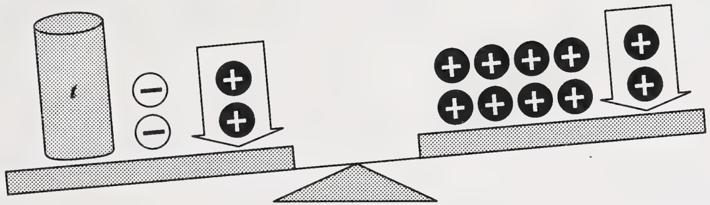


So, $r < 2$.

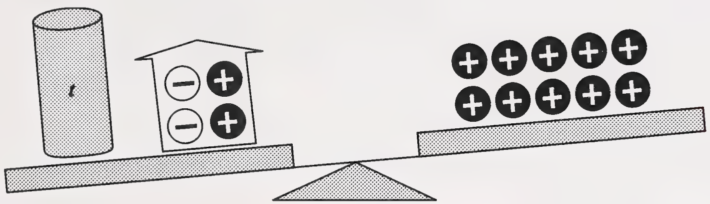
d.



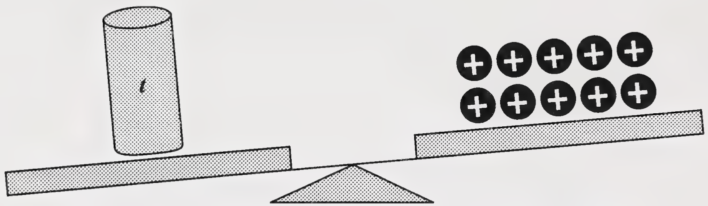
Add 2 to each side.



Simplify by removing the zeros.



The result is this.



So, $t > 10$.

2. a.

b.

c.

d.

Section 5: Practice Activity 2

1. a.

$$\begin{array}{rcl} m \cancel{+7} & < & 10 \\ \cancel{-7} & & -7 \\ \hline m & < & 3 \end{array}$$

b.

$$\begin{array}{rcl} m \cancel{-2} & > & 11 \\ \cancel{+2} & & +2 \\ \hline m & > & 13 \end{array}$$

c.

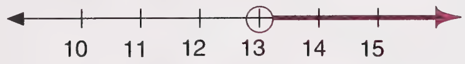
$$\begin{array}{rcl} x \cancel{+4} & < & 6 \\ \cancel{-4} & & -4 \\ \hline x & < & 2 \end{array}$$

d. $y \cancel{-3} > 8$
$\quad \cancel{+3} \quad +3$
$y \quad > 11$

e. $w \cancel{+2} > 6$
$\quad \cancel{-2} \quad -2$
$w \quad > 4$

2. a.

b.



c.

d.

e.

Section 6: Practice Activity 1

1. a. $\frac{1}{2}$ **b.** $\frac{1}{4}$ **c.** $\frac{1}{3}$ **d.** $\frac{1}{2}$

2. a.

$$2y = 18$$
$$\frac{1}{\underset{1}{\cancel{2}}} \times \overset{1}{\cancel{2}}y = \frac{1}{\underset{1}{\cancel{2}}} \times \overset{9}{\cancel{18}}$$
$$y = 9$$

LS	RS
$2y$	18
$= 2(9)$	
$= 18$	

LS = RS

b.

$$4v = 32$$
$$\frac{1}{\underset{1}{\cancel{4}}} \times \overset{1}{\cancel{4}}v = \frac{1}{\underset{1}{\cancel{4}}} \times \overset{8}{\cancel{32}}$$
$$v = 8$$

LS	RS
$4v$	32
$= 4(8)$	
$= 32$	

LS = RS

c.

$$3m = -9$$
$$\frac{1}{\underset{1}{\cancel{3}}} \times \overset{1}{\cancel{3}}m = -\left(\frac{1}{\underset{1}{\cancel{3}}} \times \overset{3}{\cancel{9}}\right)$$
$$m = -3$$

LS	RS
$3m$	-9
$= 3(-3)$	
$= -9$	

LS = RS

d.

$$2f = -4$$
$$\frac{1}{\underset{1}{\cancel{2}}} \times \overset{1}{\cancel{2}}f = -\left(\frac{1}{\underset{1}{\cancel{2}}} \times \overset{2}{\cancel{4}}\right)$$
$$f = -2$$

LS	RS
$2f$	-4
$= 2(-2)$	
$= -4$	

LS = RS

3. a.

$$-3a = 18$$
$$3a = -18$$
$$\frac{1}{\underset{1}{\cancel{3}}} \times \overset{1}{\cancel{3}}a = -\left(\frac{1}{\underset{1}{\cancel{3}}} \times \overset{6}{\cancel{18}}\right)$$
$$a = -6$$

b.

$$-5y = -10$$
$$5y = 10$$
$$\frac{1}{\underset{1}{\cancel{5}}} \times \overset{1}{\cancel{5}}y = \frac{1}{\underset{1}{\cancel{5}}} \times \overset{2}{\cancel{10}}$$
$$y = 2$$

Section 6: Practice Activity 2

1. a.

Start t → Multiply by 9 → End -72

End t ← Divide by 9 ← Start -72

So, $t = -8$.

b.

Start t → Multiply by 3 → End 30

End t ← Divide by 3 ← Start 30

So, $t = 10$.

c.

Start n → Multiply by -4 → End 44

End n ← Divide by -4 ← Start 44

So, $n = -11$.

d.

Start w → Multiply by -2 → End -10

End w ← Divide by -2 ← Start -10

So, $w = 5$.

2. a.

$$4t = 6$$

$$\frac{1}{\cancel{4}_{1}} \times \overset{1}{\cancel{4}}t = \frac{1}{\cancel{4}_{2}} \times \overset{3}{\cancel{6}}$$

$$t = \frac{3}{2}$$

LS	RS
$4t$	6
$= 4\left(\frac{3}{2}\right)$	
$= 6$	

LS = RS

b.

$$3a = \frac{1}{2}$$

$$\frac{1}{\cancel{3}_{1}} \times \overset{1}{\cancel{3}}a = \frac{1}{3} \times \frac{1}{2}$$

$$a = \frac{1}{6}$$

LS	RS
$3a$	$\frac{1}{2}$
$= 3\left(\frac{1}{6}\right)$	
$= \frac{1}{2}$	

LS = RS

c. $2r = \frac{3}{4}$

$\frac{1}{\cancel{2}_1} \times \cancel{2}^1 r = \frac{1}{2} \times \frac{3}{4}$

$r = \frac{3}{8}$

LS	RS
$2r$	$\frac{3}{4}$
$= 2\left(\frac{3}{8}\right)$	
$= \frac{3}{4}$	

LS = RS

d. $2p = 14.4$

$\frac{1}{\cancel{2}_1} \times \cancel{2}^1 p = \frac{1}{\cancel{2}_1} \times \cancel{14.4}^{7.2}$

$p = 7.2$

LS	RS
$2p$	14.4
$= 2(7.2)$	
$= 14.4$	

LS = RS

e. $3m = 0.9$

$\frac{1}{\cancel{3}_1} \times \cancel{3}^1 m = \frac{1}{\cancel{3}_1} \times \cancel{0.9}^{0.3}$

$m = 0.3$

LS	RS
$3m$	0.9
$= 3(0.3)$	
$= 0.9$	

LS = RS

3. a. $5n = 75$

$\frac{1}{\cancel{5}_1} \times \cancel{5}^1 n = \frac{1}{\cancel{5}_1} \times \cancel{75}^{15}$

$n = 15$

The number is 15.

b. $3n = -12$

$\frac{1}{\cancel{3}_1} \times \cancel{3}^1 n = -\left(\frac{1}{\cancel{3}_1} \times \cancel{12}^{4}\right)$

$n = -4$

The number is -4.

c. $2n = -56$

$\frac{1}{\cancel{2}_1} \times \cancel{2}^1 n = -\left(\frac{1}{\cancel{2}_1} \times \cancel{56}^{28}\right)$

$n = -28$

The number is -28.

d. $9n = 99$

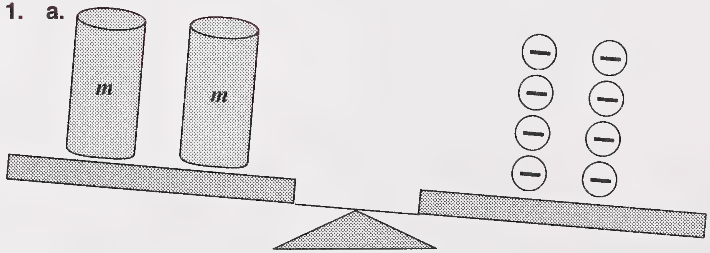
$\frac{1}{\cancel{9}_1} \times \cancel{9}^1 n = \frac{1}{\cancel{9}_1} \times \cancel{99}^{11}$

$n = 11$

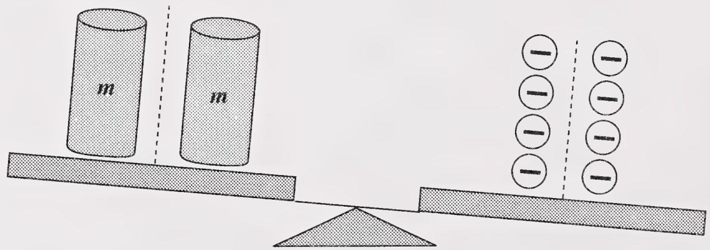
The number is 11.

Section 7: Practice Activity 1

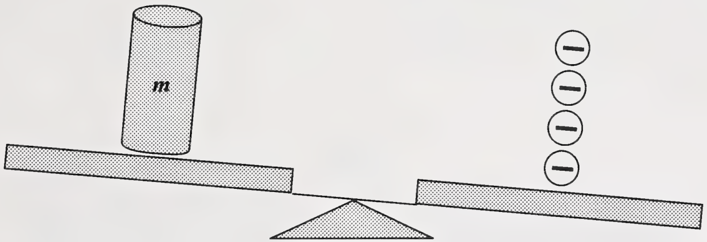
1. a.



Divide each side into two groups.

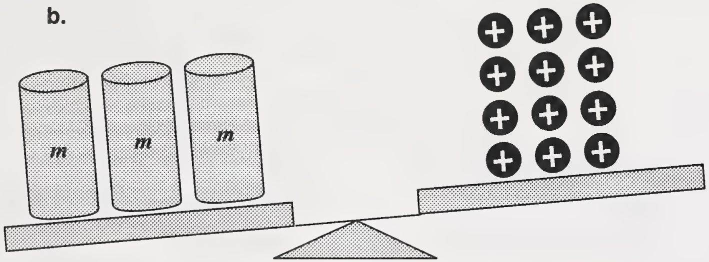


Examine one group from each side.

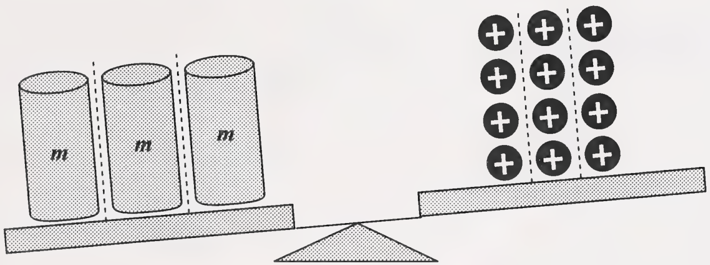


So, $m < -4$.

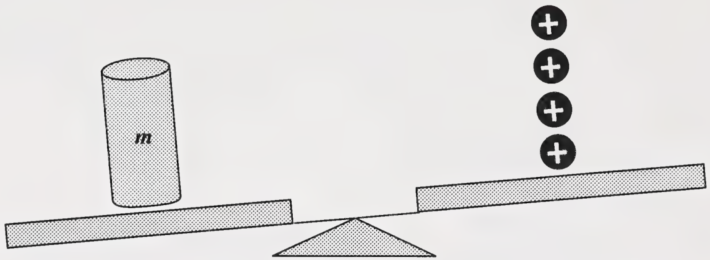
b.



Divide each side into three groups.

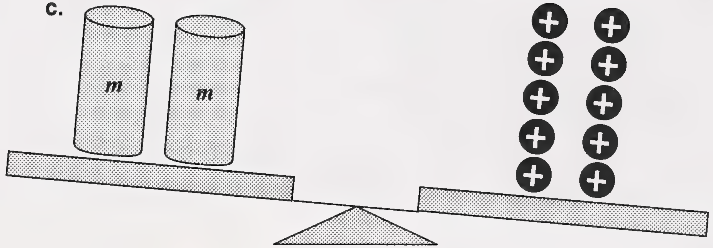


Examine one group from each side.

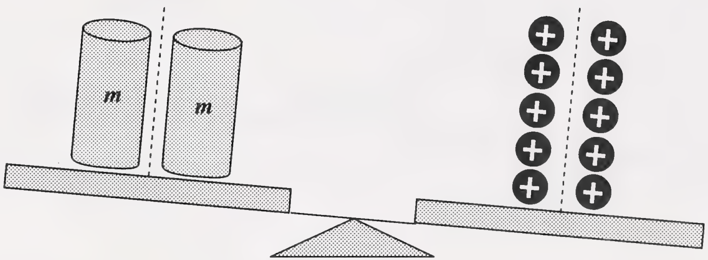


So, $m > 4$.

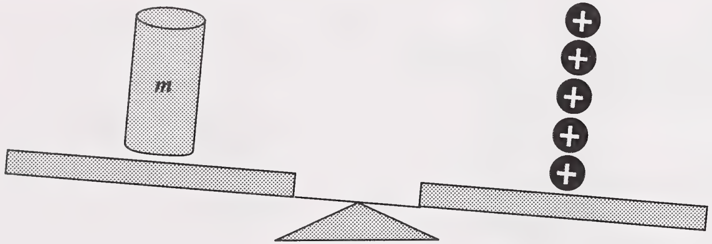
c.



Divide each side into two groups.

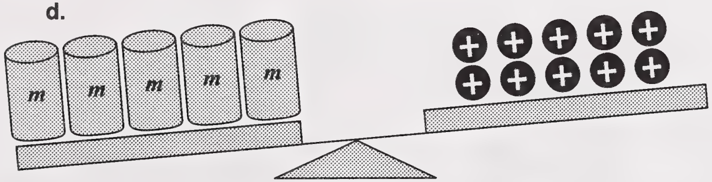


Examine one group from each side.

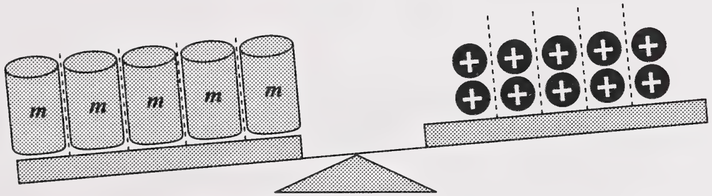


So, $m < 5$.

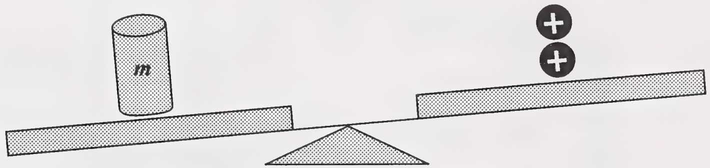
d.



Divide each side into five groups.



Examine one group from each side.

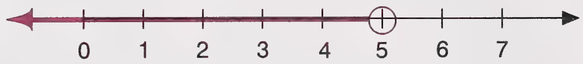


So, $m > 2$.

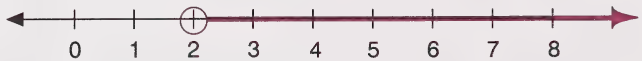
2. a.

b.

c.



d.



Section 7: Practice Activity 2

1. a.

$$4m < 12$$

$$\frac{1}{\cancel{4}_1} \times \cancel{4}^1 m < \frac{1}{\cancel{4}_1} \times \cancel{12}^3$$

$$m < 3$$

LS	RS
$4m$	12
$= 4(2)$	
$= 8$	

LS < RS

b. $3m < 27$

$$\frac{1}{\cancel{3}_1} \times \overset{1}{\cancel{3}}m < \frac{1}{\cancel{3}_1} \times \overset{9}{\cancel{27}}$$

$$m < 9$$

LS	RS
$3m$	27
$= 3(8)$	
$= 24$	

LS < RS

c. $2m > 18$

$$\frac{1}{\cancel{2}_1} \times \overset{1}{\cancel{2}}m > \frac{1}{\cancel{2}_1} \times \overset{9}{\cancel{18}}$$

$$m > 9$$

LS	RS
$2m$	18
$= 2(10)$	
$= 20$	

LS > RS

d. $5m > 25$

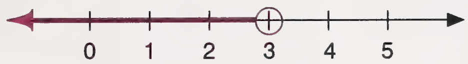
$$\frac{1}{\cancel{5}_1} \times \overset{1}{\cancel{5}}m > \frac{1}{\cancel{5}_1} \times \overset{5}{\cancel{25}}$$

$$m > 5$$

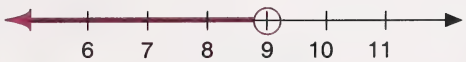
LS	RS
$5m$	25
$= 5(6)$	
$= 30$	

LS > RS

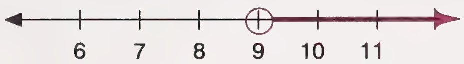
2. a.



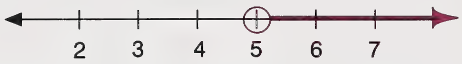
b.



c.

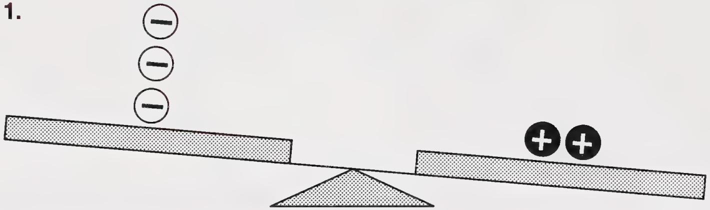


d.

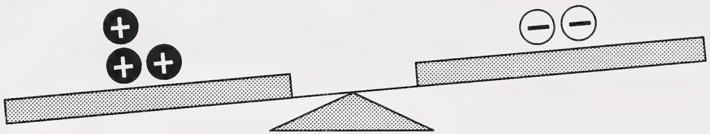


Section 7: Practice Activity 3

1.

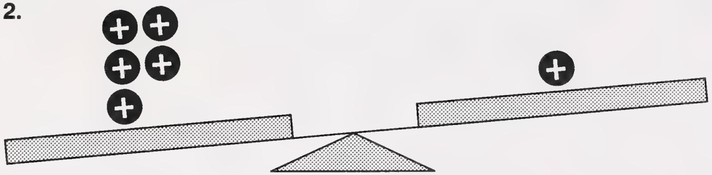


If you multiply each side by −1, the inequality changes.

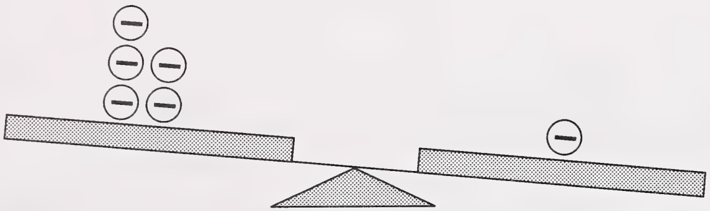


$+3 > -2$

2.

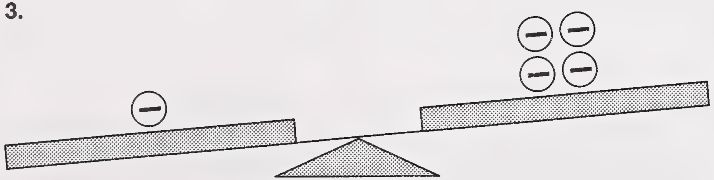


If you multiply each side by −1, the inequality changes.

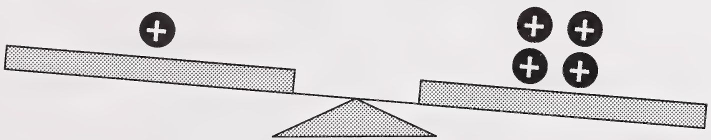


$-5 < -1$

3.



If you multiply each side by -1, the inequality changes.



$1 < 4$

Section 8: Practice Activity 1

1. a. 5 b. 11 c. 2 d. 8 e. 7

2. a.
$$\frac{n}{5} = 6$$
$$\overset{1}{\cancel{5}} \times \frac{n}{\underset{1}{\cancel{5}}} = 5 \times 6$$
$$n = 30$$

b.
$$\frac{a}{11} = 3$$
$$\overset{1}{\cancel{11}} \times \frac{a}{\underset{1}{\cancel{11}}} = 11 \times 3$$
$$a = 33$$

c.
$$\frac{r}{2} = -12$$
$$\overset{1}{\cancel{2}} \times \frac{r}{\underset{1}{\cancel{2}}} = 2 \times (-12)$$
$$r = -24$$

d.
$$\frac{d}{8} = 2$$
$$\overset{1}{\cancel{8}} \times \frac{d}{\underset{1}{\cancel{8}}} = 8 \times 2$$
$$d = 16$$

e.
$$\frac{b}{7} = -1$$
$$\overset{1}{\cancel{7}} \times \frac{b}{\underset{1}{\cancel{7}}} = 7 \times (-1)$$
$$b = -7$$

Section 8: Practice Activity 2

1. a. 15 b. 9 c. 5 d. 7 e. 20

2. a.
$$\frac{n}{15} = \frac{2}{3}$$
$$\overset{1}{\cancel{15}} \times \frac{n}{\underset{1}{\cancel{15}}} = \overset{5}{\cancel{15}} \times \frac{2}{\underset{1}{\cancel{3}}}$$
$$n = 10$$

b.
$$\frac{f}{9} = \frac{8}{3}$$
$$\overset{1}{\cancel{9}} \times \frac{f}{\underset{1}{\cancel{9}}} = \overset{3}{\cancel{9}} \times \frac{8}{\underset{1}{\cancel{3}}}$$
$$f = 24$$

c. $\frac{y}{5} = \frac{5}{6}$

$\cancel{5}^{1} \times \frac{y}{\cancel{5}_{1}} = 5 \times \frac{5}{6}$

$y = \frac{25}{6}$ or $4\frac{1}{6}$

d. $\frac{r}{7} = \frac{5}{8}$

$\cancel{7}^{1} \times \frac{r}{\cancel{7}_{1}} = 7 \times \frac{5}{8}$

$r = \frac{35}{8}$ or $4\frac{3}{8}$

e. $\frac{3}{4} = \frac{c}{20}$

$\cancel{20}^{5} \times \frac{3}{\cancel{4}_{1}} = \cancel{20}^{1} \times \frac{c}{\cancel{20}_{1}}$

$15 = c$

Section 8: Practice Activity 3

1. a.

Start a → Divide by 3 → End 2

End a ← Multiply by 3 ← Start 2

So, $a = 6$.

b.

Start b → Divide by 4 → End 16

End b ← Multiply by 4 ← Start 16

So, $b = 64$.

c.

Start c → Divide by 2 → End $\frac{3}{8}$

End c ← Multiply by 2 ← Start $\frac{3}{8}$

So, $c = \frac{3}{4}$.

d.

Start d → Divide by 5 → End $\frac{8}{25}$

End d ← Multiply by 5 ← Start $\frac{8}{25}$

So, $d = \frac{8}{5}$ or $1\frac{3}{5}$.

2. a. $\frac{1}{8}n = 5$

$\overset{1}{\cancel{8}} \times \frac{1}{\underset{1}{\cancel{8}}}n = 8 \times 5$

$n = 40$

b. $\frac{1}{5}n = -2$

$\overset{1}{\cancel{5}} \times \frac{1}{\underset{1}{\cancel{5}}}n = 5 \times (-2)$

$n = -10$

c. $\frac{1}{4}n = \frac{1}{2}$

$\overset{1}{\cancel{4}} \times \frac{1}{\underset{1}{\cancel{4}}}n = \overset{2}{\cancel{4}} \times \frac{1}{\underset{1}{\cancel{2}}}$

$n = 2$

d. $\frac{1}{2}n = \frac{3}{4}$

$\overset{1}{\cancel{2}} \times \frac{1}{\underset{1}{\cancel{2}}}n = \overset{1}{\cancel{2}} \times \frac{3}{\underset{2}{\cancel{4}}}$

$n = \frac{3}{2}$ or $1\frac{1}{2}$

Section 8: Practice Activity 4

1. a. $\frac{n}{5} = \frac{3}{4}$

$\overset{4}{\cancel{20}} \times \frac{n}{\underset{1}{\cancel{5}}} = \overset{5}{\cancel{20}} \times \frac{3}{\underset{1}{\cancel{4}}}$

$4n = 15$

$n = \frac{15}{4}$

So, $n = \frac{15}{4}$ or 3.75.

b. $\frac{2}{n} = \frac{5}{8}$

$\overset{1}{8\cancel{n}} \times \frac{2}{\underset{1}{\cancel{n}}} = \overset{1}{\cancel{8}}n \times \frac{5}{\underset{1}{\cancel{8}}}$

$16 = 5n$

$\frac{16}{5} = n$

So, $n = \frac{16}{5}$ or 3.2.

c. $\frac{x}{3} = \frac{4}{5}$

$\overset{3}{\cancel{15}} \times \frac{x}{\underset{1}{\cancel{3}}} = \overset{3}{\cancel{15}} \times \frac{4}{\underset{1}{\cancel{5}}}$

$5x = 12$

$x = \frac{12}{5}$

So, $x = \frac{12}{5}$ or 2.4.

d. $\frac{3}{r} = \frac{5}{8}$

$\overset{1}{8\cancel{r}} \times \frac{3}{\underset{1}{\cancel{r}}} = \overset{1}{\cancel{8}}r \times \frac{5}{\underset{1}{\cancel{8}}}$

$24 = 5r$

$\frac{24}{5} = r$

So, $r = \frac{24}{5}$ or 4.8.

2. a. $\frac{n}{5} \times \frac{3}{4}$

$4n = 15$

$n = \frac{15}{4}$

So, $n = \frac{15}{4}$ or 3.75.

b. $\frac{2}{n} \times \frac{5}{8}$

$16 = 5n$

$\frac{16}{5} = n$

So, $n = \frac{16}{5}$ or 3.2.

c. $\frac{x}{3} \times \frac{4}{5}$

$5x = 12$

$x = \frac{12}{5}$

So, $x = \frac{12}{5}$ or 2.4.

d. $\frac{3}{r} \times \frac{5}{8}$

$24 = 5r$

$\frac{24}{5} = r$

So, $r = \frac{24}{5}$ or 4.8.

3. a.

Key Press	Display
3 × 5 ÷ 4 =	3.75

b.

Key Press	Display
2 × 8 ÷ 5 =	3.2

c.

Key Press	Display
3 × 4 ÷ 5 =	2.4

d.

Key Press	Display
3 × 8 ÷ 5 =	4.8

Section 9: Practice Activity 1

Computer Alternative

1. computer corrected

Print Alternative

2. a.
$$\begin{aligned} 5x \cancel{+6} &= 31 \\ \cancel{-6} &\quad -6 \\ \hline 5x &= 25 \\ x &= 5 \end{aligned}$$

b.
$$\begin{aligned} 6x \cancel{-3} &= 15 \\ \cancel{+3} &\quad +3 \\ \hline 6x &= 18 \\ x &= 3 \end{aligned}$$

c.
$$\begin{aligned} 8a \cancel{+6} &= 22 \\ \cancel{-6} &\quad -6 \\ \hline 8a &= 16 \\ a &= 2 \end{aligned}$$

d.
$$\begin{aligned} 3c \cancel{-3} &= -24 \\ \cancel{+3} &\quad +3 \\ \hline 3c &= -21 \\ c &= -7 \end{aligned}$$

e.
$$\begin{aligned} 11n \cancel{+44} &= 0 \\ \cancel{-44} &\quad -44 \\ \hline 11n &= -44 \\ n &= -4 \end{aligned}$$

Section 9: Practice Activity 2

1. a.

Start a → Multiply by 3 → Subtract 2 → End 7

End a ← Divide by 3 ← Add 2 ← Start 7

So, $a = 3$.

b.

Start b → Multiply by 2 → Add 1 → End −9

End b ← Divide by 2 ← Subtract 1 ← Start −9

So, $b = -5$.

c.

$\boxed{\text{Start } c} \rightarrow \boxed{\text{Multiply by 3}} \rightarrow \boxed{\text{Subtract 4}} \rightarrow \boxed{\text{End } 5}$

$\boxed{\text{End } c} \leftarrow \boxed{\text{Divide by 3}} \leftarrow \boxed{\text{Add 4}} \leftarrow \boxed{\text{Start } 5}$

So, $c = 3$.

d.

$\boxed{\text{Start } b} \rightarrow \boxed{\text{Multiply by 4}} \rightarrow \boxed{\text{Subtract 1}} \rightarrow \boxed{\text{End } -9}$

$\boxed{\text{End } b} \leftarrow \boxed{\text{Divide by 4}} \leftarrow \boxed{\text{Add 1}} \leftarrow \boxed{\text{Start } -9}$

So, $b = -2$.

e.

$\boxed{\text{Start } d} \rightarrow \boxed{\text{Multiply by 2}} \rightarrow \boxed{\text{Add 1}} \rightarrow \boxed{\text{End } 5}$

$\boxed{\text{End } d} \leftarrow \boxed{\text{Divide by 2}} \leftarrow \boxed{\text{Subtract 1}} \leftarrow \boxed{\text{Start } 5}$

So, $d = 2$.

2. a.

$$\begin{array}{rcl} 2a \cancel{+\frac{1}{2}} & = & \frac{3}{4} \\ \cancel{-\frac{1}{2}} & & -\frac{1}{2} \\ \hline 2a & = & \frac{1}{4} \\ a & = & \frac{1}{8} \end{array}$$

b.

$$\begin{array}{rcl} 3b \cancel{-1} & = & 8.3 \\ \cancel{+1} & & +1 \\ \hline 3b & = & 9.3 \\ b & = & 3.1 \end{array}$$

c.

$$\begin{array}{rcl} 5c \cancel{+1.5} & = & -10 \\ \cancel{-1.5} & & -1.5 \\ \hline 5c & = & -11.5 \\ c & = & -2.3 \end{array}$$

3. LOOKATTHEORANGEMAMALAID
(Look at the orange mama laid.)

Section 10: Practice Activity 1

1. a.

$$\begin{aligned} 2x + 4x &= 12 \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

b.

$$\begin{aligned} 6y - 2y &= -8 \\ 4y &= -8 \\ y &= -2 \end{aligned}$$

c.

$$\begin{aligned} a - 3a &= 6 \\ -2a &= 6 \\ 2a &= -6 \\ a &= -3 \end{aligned}$$

2. a.

$$\begin{array}{rcl} 3(y-2) & = & 6 \\ 3y \cancel{-6} & = & 6 \\ \cancel{+6} & & +6 \\ \hline 3y & = & 12 \\ y & = & 4 \end{array}$$

b.

$$\begin{array}{rcl} 2(x+1) & = & -6 \\ 2x \cancel{+2} & = & -6 \\ \cancel{-2} & & -2 \\ \hline 2x & = & -8 \\ x & = & -4 \end{array}$$

c. $$\begin{aligned} 3(7-4x) &= 29 \\ \cancel{21}-12x &= 29 \\ \cancel{-21} \quad & -21 \\ \hline -12x &= 8 \\ 12x &= -8 \\ x &= -\frac{8}{12} \text{ or } -\frac{2}{3} \end{aligned}$$

3. ILOVEMATH
(I love math.)

Section 10: Practice Activity 2

ITSTWIRLY
(It's twirly.)

The calculations for the puzzle are shown because of their complexity.

1. Let n be the first number.
Let $4n$ be the second number.

$$\begin{aligned} n+4n &= 45 \\ 5n &= 45 \\ n &= 9 \end{aligned}$$

If $n = 9$, then $4n = 4 \times 9 = 36$

The numbers are 9 and 36.

2. Let n be the lesser number.
Let $3n$ be the greater number.

$$\begin{aligned} n+3n &= 44 \\ 4n &= 44 \\ n &= 11 \end{aligned}$$

If $n = 11$, then $3n = 3 \times 11 = 33$

The numbers are 11 and 33.

3. Let n be the first number.
Let $n+7$ be the second number.

$$\begin{aligned} n+n+7 &= 47 \\ 2n+\cancel{7} &= 47 \\ \cancel{-7} \quad & -7 \\ \hline 2n &= 40 \\ n &= 20 \end{aligned}$$

If $n = 20$, then $n+7 = 20+7 = 27$

The numbers are 20 and 27.

4. Let n be the lesser number.
Let $n+10$ be the greater number.

$$\begin{aligned} n+n+10 &= 38 \\ 2n+\cancel{10} &= 38 \\ \cancel{-10} \quad & -10 \\ \hline 2n &= 28 \\ n &= 14 \end{aligned}$$

If $n = 14$, then $n+10 = 14+10 = 24$

The numbers are 14 and 24.

5. Let n be the second number.
Let $n-5$ be the first number.

$$\begin{aligned} n+n-5 &= 31 \\ 2n-\cancel{5} &= 31 \\ \cancel{+5} \quad & +5 \\ \hline 2n &= 36 \\ n &= 18 \end{aligned}$$

If $n = 18$, then $n-5 = 18-5 = 13$

The numbers are 13 and 18.

6. Let n be the first number.
Let $2n+1$ be the second number.

$$\begin{aligned} n+2n+1 &= 25 \\ 3n\cancel{+1} &= 25 \\ \cancel{-1}\quad &\ -1 \\ \hline 3n &= 24 \\ n &= 8 \end{aligned}$$

If $n=8$, then $2n+1 = 2\times 8+1$
$= 16+1$
$= 17$

The numbers are 8 and 17.

7. Let n be the lesser number.
Let $5n+8$ be the greater number.

$$\begin{aligned} n+5n+8 &= 68 \\ 6n\cancel{+8} &= 68 \\ \cancel{-8}\quad &\ -8 \\ \hline 6n &= 60 \\ n &= 10 \end{aligned}$$

If $n=10$, then $5n+8 = 5\times 10+8$
$= 50+8$
$= 58$

The numbers are 10 and 58.

8. Let n be the first number.
Let $2n-3$ be the second number.

$$\begin{aligned} n+2n-3 &= 42 \\ 3n\cancel{-3} &= 42 \\ \cancel{+3}\quad &\ +3 \\ \hline 3n &= 45 \\ n &= 15 \end{aligned}$$

If $n=15$, then $2n-3 = 2\times 15-3$
$= 30-3$
$= 27$

The numbers are 15 and 27.

9. Let n be the first number.
Let $4n-2$ be the second number.

$$\begin{aligned} n+4n-2 &= 33 \\ 5n\cancel{-2} &= 33 \\ \cancel{+2}\quad &\ +2 \\ \hline 5n &= 35 \\ n &= 7 \end{aligned}$$

If $n=7$, then $4n-2 = 4\times 7-2$
$= 28-2$
$= 26$

The numbers are 7 and 26.

10. Let n be the number of baskets.
Let $n+6$ be the number of missed shots.

$$\begin{aligned} n+n+6 &= 70 \\ 2n\cancel{+6} &= 70 \\ \cancel{-6}\quad &\ -6 \\ \hline 2n &= 64 \\ n &= 32 \end{aligned}$$

The player made 32 baskets.

11. Let n be the length of the advertising portion.
Let $4n$ be the length of the entertainment portion.

$$\begin{aligned} n+4n &= 60 \\ 5n &= 60 \\ n &= 12 \end{aligned}$$

The advertising portion was 12 min long.

Section 11: Practice Activity

1. a.
$$\begin{array}{rcl} 5n\cancel{-8} &=& 3n \\ \cancel{+8} && +8 \\ \hline 5n &=& \cancel{3n}+8 \\ -3n && \cancel{-3n} \\ \hline 2n &=& 8 \\ n &=& 4 \end{array}$$

b.
$$\begin{array}{rcl} \cancel{10}-n &=& -6n \\ \cancel{-10} && -10 \\ \hline -n &=& \cancel{-6n}-10 \\ +6n && \cancel{+6n} \\ \hline 5n &=& -10 \\ n &=& -2 \end{array}$$

c.
$$\begin{array}{rcl} 3x\cancel{+9} &=& 2x-5 \\ \cancel{-9} && -9 \\ \hline 3x &=& \cancel{2x}-14 \\ -2x && \cancel{-2x} \\ \hline x &=& -14 \end{array}$$

d.
$$\begin{array}{rcl} 4-8a &=& 2a-66 \\ -4 && -4 \\ \hline -8a &=& \cancel{2a}-70 \\ -2a && \cancel{-2a} \\ \hline -10a &=& -70 \\ 10a &=& 70 \\ a &=& 7 \end{array}$$

e.
$$\begin{array}{rcl} \cancel{3}+2a &=& -5a-11 \\ \cancel{-3} && -3 \\ \hline 2a &=& \cancel{-5a}-14 \\ +5a && \cancel{+5a} \\ \hline 7a &=& -14 \\ a &=& -2 \end{array}$$

2. *Tragedy on the Cliff* by EILEEN DOVER
Mystery of the Creaking Door by RUSTY HINGES
P.S. by ADALINE MOORE

Section 12: Practice Activity

1. a.
$$\begin{array}{rcl} x^2\cancel{+1} &=& 10 \\ \cancel{-1} && -1 \\ \hline x^2 &=& 9 \\ x &=& 3 \end{array}$$

b.
$$\begin{array}{rcl} r^2\cancel{-1} &=& 24 \\ \cancel{+1} && +1 \\ \hline r^2 &=& 25 \\ r &=& 5 \end{array}$$

c.
$$\begin{array}{c} 3t^2 = 108 \\ t^2 = 36 \\ t = 6 \end{array}$$

d.
$$\begin{array}{c} -2n^2 = -128 \\ 2n^2 = 128 \\ n^2 = 64 \\ n = 8 \end{array}$$

e.
$$\begin{array}{rcl} \sqrt{s}\cancel{+2} &=& 8 \\ \cancel{-2} && -2 \\ \hline \sqrt{s} &=& 6 \\ s &=& 36 \end{array}$$

f.
$$\begin{array}{c} 5\sqrt{n} = 10 \\ \sqrt{n} = 2 \\ n = 4 \end{array}$$

2. a.
$$\begin{array}{rcl} \sqrt{n}\cancel{+3} &=& 12 \\ \cancel{-3} && -3 \\ \hline \sqrt{n} &=& 9 \\ n &=& 81 \end{array}$$

The number is 81.

b.
$$\begin{array}{rcl} n^2\cancel{-8} &=& 17 \\ \cancel{+8} && +8 \\ \hline n^2 &=& 25 \\ n &=& 5 \end{array}$$

The number is 5.

c.
$$\begin{array}{c} 10n^2 = 40 \\ n^2 = 4 \\ n = 2 \end{array}$$

The number is 2.

d.
$$\begin{array}{c} 3\sqrt{n} = 15 \\ \sqrt{n} = 5 \\ n = 25 \end{array}$$

The number is 25.

Section 13: Practice Activity

Note: Any method can be used to solve the problems. The equation-solving method is shown.

1. **a.** Let the number of quarters be n.
Let the number of dimes be $2n$.

$$\begin{aligned} n + 2n &= 9 \\ 3n &= 9 \\ n &= 3 \end{aligned}$$

If $n = 3$, then $2n = 2 \times 3$
$= 6$

Lori has 6 dimes.

b.

	Present Age	Age in 8 Years
Mrs. Chan	c	$c+8$
Mrs. Wong	$c+9$	$c+17$

$$\begin{aligned} 7(c+8) &= 6(c+17) \\ 7c + \cancel{56} &= 6c + 102 \\ \cancel{-56} & \quad -56 \\ 7c &= \cancel{6c} + 46 \\ -6c & \quad \cancel{-6c} \\ c &= 46 \end{aligned}$$

If $c = 46$, then $c + 9 = 46 + 9$
$= 55$

Mrs. Chan is 46 years old and Mrs. Wong is 55 years old.

c. Let the price of the running shoes be r.
Let the price of the track shoes be $2r$.

$$\begin{aligned} 2 \times 2r + 2r &= 330 \\ 4r + 2r &= 330 \\ 6r &= 330 \\ r &= 55 \end{aligned}$$

If $r = 55$, then $2r = 110$.

The running shoes cost \$55. The track shoes cost \$110.

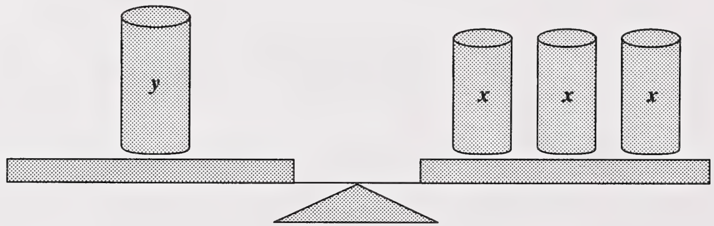
2. SHEPASSEDTHEBUCK
(She passed the buck.)

3. A LUNA TICK

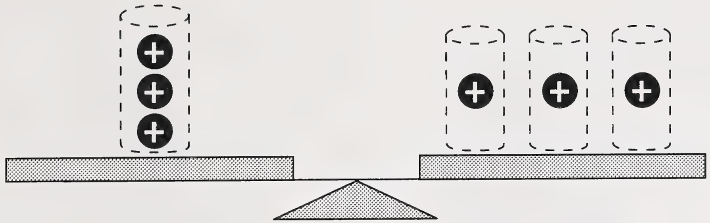
Section 14: Practice Activity 1

1. What values for x and y will make $y = 3x$ a true statement?

Model the equation.



By inspection it is clear that the ordered pair $(1, 3)$ is a solution. Verify the solution.

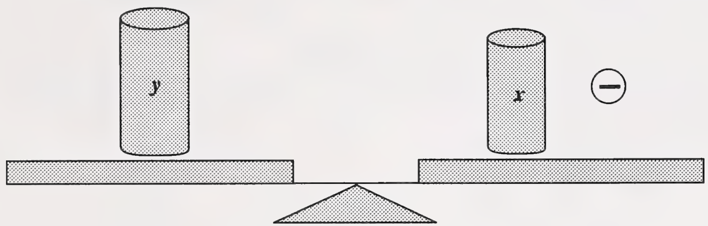


Each side of the equation equals 3. The scale is balanced. The values $(1, 3)$ make the statement true.

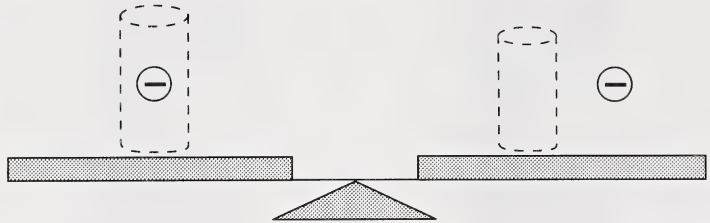
Other ordered pairs that make this equation true are $(0, 0)$, $(-1, -3)$, $(-3, -9)$, $(2, 6)$, $(3, 9)$, and so on.

2. What values for x and y will make $y = x - 1$ a true statement?

Model the equation.



By inspection it is clear that the ordered pair $(0, -1)$ is a solution. Verify the solution.

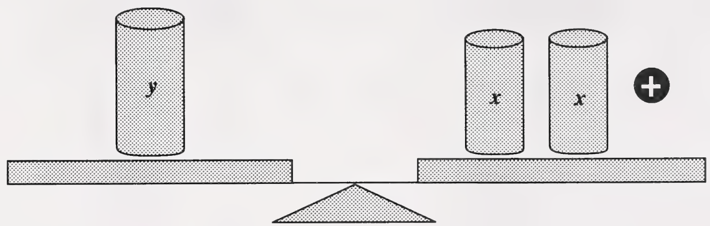


Each side equals -1, so the equation is balanced. The values $(0, -1)$ make the equation true.

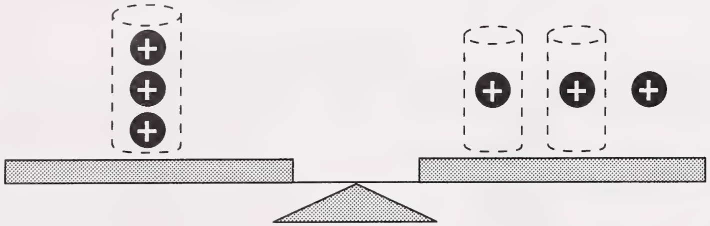
Other ordered pairs that make the equation true are $(1, 0)$, $(-2, -3)$, $(3, 2)$, $(4, 3)$, $(10, 9)$, and so on.

3. What values for x and y will make $y = 2x + 1$ a true statement?

Model the equation.



By inspection it is clear that $(1, 3)$ is a solution. Verify the solution.



Each side equals 3. The equation is balanced, so $(1, 3)$ are true values.

Other ordered pairs that make the statement true are $(-2, -3)$, $(-1, -1)$, $(0, 1)$, $(2, 5)$, $(3, 7)$, $(4, 9)$, and so on.

Section 14: Practice Activity 2

1. a.

$y = -2x$	
x	y
1	−2
4	−8
−5	10
3	−6

b.

$y = 2x + 4$	
x	y
3	10
−7	−10
1	6
−3	−2

c.

$y = -3x + 1$	
x	y
3	−8
−3	10
4	−11
−2	7

d.

$y = \frac{1}{2}x - 4$	
x	y
10	1
−2	−5
4	−2
−8	−8

e.

$y = -x + 6$	
x	y
4	2
−1	7
6	0
0	6

f.

$y = -\frac{3}{2}x - 2$	
x	y
4	−8
2	−5
0	−2
−2	1

g.

$y = -3x + 7$	
x	y
6	−11
1	4
0	7
−2	13

h.

$y = -x + 1$	
x	y
−2	3
−9	10
9	−8
6	−5

Section 14: Practice Activity 3

1. a.

$y = x + 2$		
x	y	Pattern
0	2	
1	3	+1
2	4	+1
3	5	+1
4	6	+1
5	7	+1

b.

$y = 4x - 1$		
x	y	Pattern
0	−1	
1	3	+4
2	7	+4
3	11	+4
4	15	+4
5	19	+4

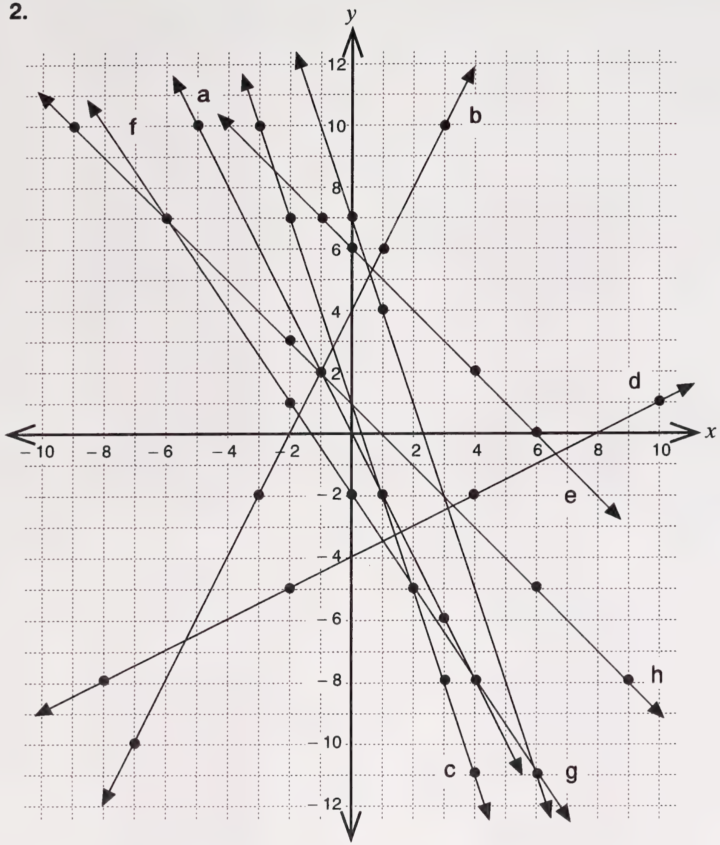
c.

$y = 2x$		
x	y	Pattern
0	0	
1	2	+2
2	4	+2
3	6	+2
4	8	+2
5	10	+2

d.

$y = 3x + 2$		
x	y	Pattern
0	2	
1	5	+3
2	8	+3
3	11	+3
4	14	+3
5	17	+3

2.



3. All the graphs are straight lines. The graphs of equations with positive coefficients slant from top right to bottom left. The graphs of equations with negative coefficients slant from top left to bottom right.

2. In Question 1 the number in the pattern is also the numerical coefficient of x.

3. a. $y = \square\, x - 3$

x	y
0	–3
1	1
2	5
3	9
4	13

Pattern: +4, +4, +4, +4

So, the equation is

$y = \boxed{4}\, x - 3.$

b. $y = \square\, x + 1$

x	y
0	1
1	7
2	13
3	19
4	25

Pattern: +6, +6, +6, +6

So, the equation is

$y = \boxed{6}\, x + 1.$

c. $y = \square\, x - 1$

x	y
0	–1
1	1
2	3
3	5
4	7

Pattern: +2, +2, +2, +2

So, the equation is

$y = \boxed{2}\, x - 1.$

d. $y = \square\, x + 4$

x	y
0	4
1	7
2	10
3	13
4	16

Pattern: +3, +3, +3, +3

So, the equation is

$y = \boxed{3}\, x + 4.$

e. $y = \square\, x + \square$

x	y
0	3
1	5
2	7
3	9
4	11

Pattern: +2, +2, +2, +2

So, the equation is

$y = \boxed{2}\, x + \boxed{3}.$

f. $y = \square\, x - \square$

x	y
0	–4
1	–1
2	2
3	5
4	8

Pattern: +3, +3, +3, +3

So, the equation is

$y = \boxed{3}\, x - \boxed{4}.$

g. $y = \square\, x + \square$

x	y
0	1
1	3
2	5
3	7
4	9

Pattern: +2, +2, +2, +2

So, the equation is

$y = \boxed{2}\, x + \boxed{1}.$

h. $y = \square$

x	y
0	–3
1	2
2	7
3	12
4	17

Pattern: +5, +5, +5, +5

So, the equation is

$y = \boxed{5x - 3}.$

i.

$y =$	
x	y
0	1
1	4
2	7
3	10
4	13

Pattern

+3
+3
+3
+3

So, the equation is

$y = \boxed{3x+1}$.

Section 14: Practice Activity 4

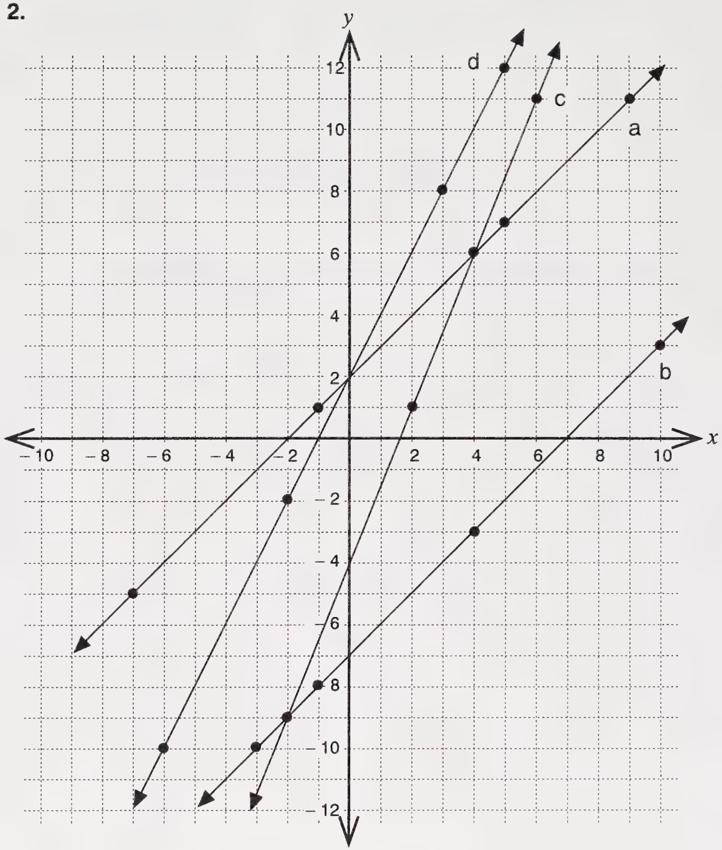
1. a.
$$\begin{aligned} y \cancel{-x} &= 2 \\ \cancel{+x} \quad &\quad +x \\ \hline y &= x+2 \end{aligned}$$

b.
$$\begin{aligned} y \cancel{+3} &= x-4 \\ \cancel{-3} \quad &\quad -3 \\ \hline y &= x-7 \end{aligned}$$

c.
$$\begin{aligned} 2y &= 5x-8 \\ \frac{2y}{2} &= \frac{5x}{2} - \frac{8}{2} \\ y &= \frac{5x}{2} - 4 \end{aligned}$$

d.
$$\begin{aligned} 3y \cancel{+1} &= 6x+7 \\ \cancel{-1} \quad &\quad -1 \\ \hline 3y &= 6x+6 \\ \frac{3y}{3} &= \frac{6x}{3} + \frac{6}{3} \\ y &= 2x+2 \end{aligned}$$

2.



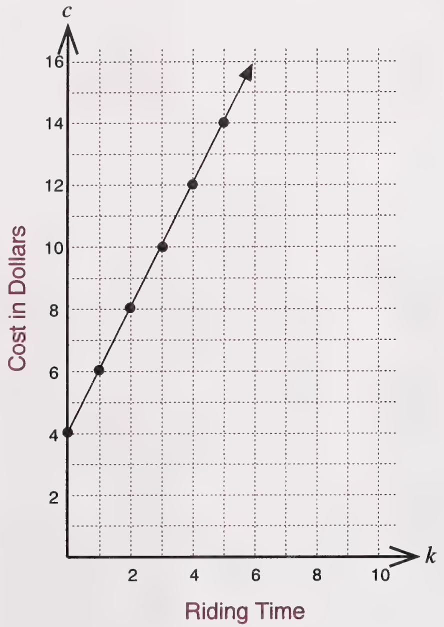
Section 15: Practice Activity 1

1. a. The cost equals two times the riding time plus four.

   b. $c = 4 + 2t$

   c. $(1, 6)$, $(2, 8)$, $(3, 10)$, $(4, 12)$, $(5, 14)$, …

   d.

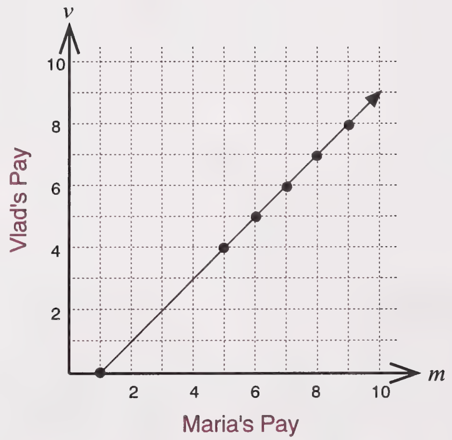


2. a. Vlad's pay equals Maria's minus one dollar.

   b. $v = m - 1$

   c. $(1, 0)$, $(5, 4)$, $(6, 5)$, $(7, 6)$

   d.



Section 15: Practice Activity 2

1. a. $A = P + I$

   $= 1000 + 90$

   $= 1090$

   The amount paid to the lender is \$1090.

b. Method 1

$$\begin{aligned} A &= P + I \\ 1620 &= P \cancel{+120} \\ -120 & \quad \cancel{-120} \\ \hline 1500 &= P \end{aligned}$$

The principal is \$1500.

Method 2

$$\begin{aligned} A &= P \cancel{+I} \\ -I & \quad \cancel{-I} \\ \hline A - I &= P \end{aligned}$$

So, $P = A - I$

$= 1620 - 120$

$= 1500$

The principal is \$1500.

2. a. $d = 343t$

$= 343 \times 6$

$= 2058$

Fatima is 2058 m or 2.058 km from the storm.

b. Method 1

$$d = 343t$$

$$\frac{d}{343} = \frac{343t}{343}$$

$$\frac{d}{343} = t$$

So, $t = \frac{d}{343}$

$= \frac{5000}{343}$

$\doteq 15$

5 km = 5000 m

It will be about 15 s between the time he sees the lightning and hears the thunder.

Method 2

$$d = 343t$$

$$5000 = 343t$$

$$\frac{5000}{343} = \frac{343t}{343}$$

$$15 \doteq t$$

5 km = 5000 m

It will be about 15 s between the time he sees the lightning and hears the thunder.

3. a. $t = \frac{n}{8} + 5$

$= \frac{80}{8} + 5$

$= 10 + 5$

$= 15$

The temperature is about 15°C.

b. Method 1

$$\begin{aligned} t &= \frac{n}{8} + 5 \\ 25 &= \frac{n}{8} \cancel{+5} \\ -5 & \quad \cancel{-5} \\ \hline 20 &= \frac{n}{8} \\ 160 &= n \end{aligned}$$

At 25°C a cricket should chirp about 160 times in one minute.

Method 2

$$t = \frac{n}{8} \cancel{+5}$$
$$-5 \qquad \cancel{-5}$$
$$t - 5 = \frac{n}{8}$$
$$8(t-5) = \overset{1}{\cancel{8}} \times \frac{n}{\underset{1}{\cancel{8}}}$$
$$8t - 40 = n$$

So, $n = 8t - 40$

$n = 8t - 40$

$= 8 \times 25 - 40$

$= 200 - 40$

$= 160$

At 25°C a cricket should chirp about 160 times in one minute.

4. a. $d = \frac{v^2}{210}$

$= \frac{80^2}{210}$

$= \frac{6400}{210}$

$\doteq 30.5$

The breaking distance is about 30.5 m.

b. $d = \frac{v^2}{210}$

$62 = \frac{v^2}{210}$

$210 \times 62 = \overset{1}{\cancel{210}} \times \frac{v^2}{\underset{1}{\cancel{210}}}$

$13020 = v^2$

$114 \doteq v$

The speed of the driver was about 114 km/h.

5. a. $t = 0.2\sqrt{\ell}$

$= 0.2\sqrt{64}$

$= 0.2 \times 8$

$= 1.6$

The time required for the swing is 1.6 s.

b. $t = 0.2\sqrt{\ell}$

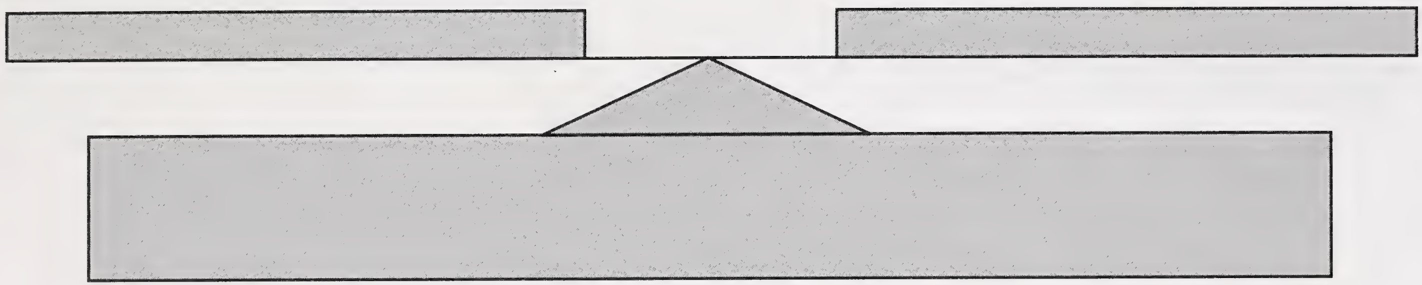
$2 = 0.2\sqrt{\ell}$

$10 = \sqrt{\ell}$

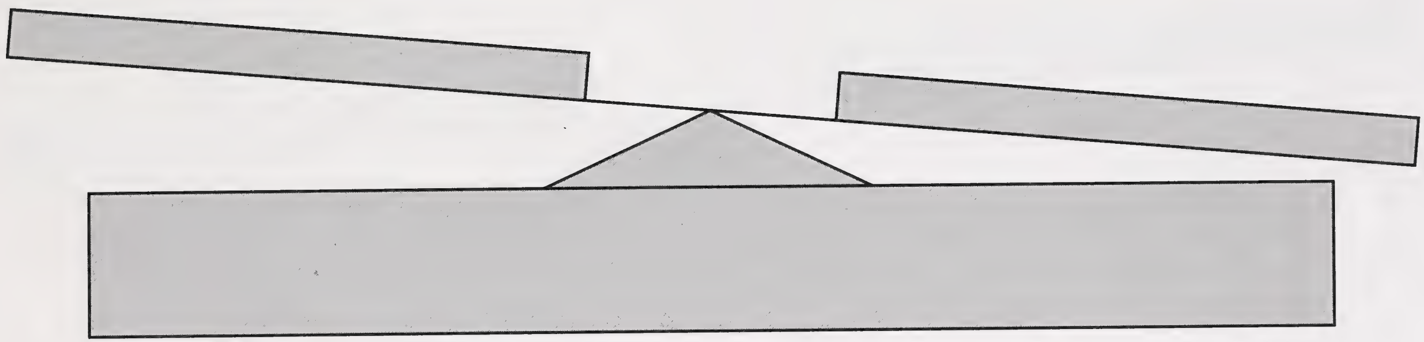
$100 = \ell$

The length of the pendulum would be 100 cm.

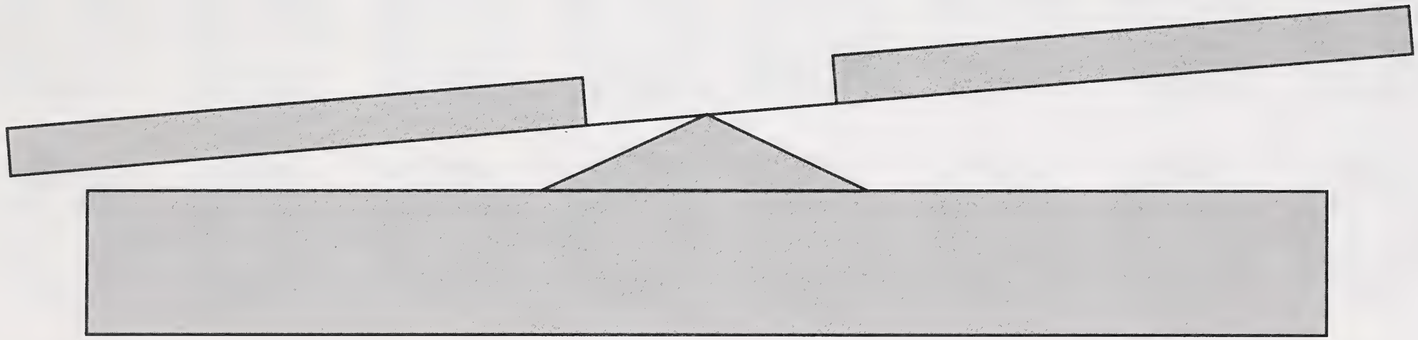
Equation Scale



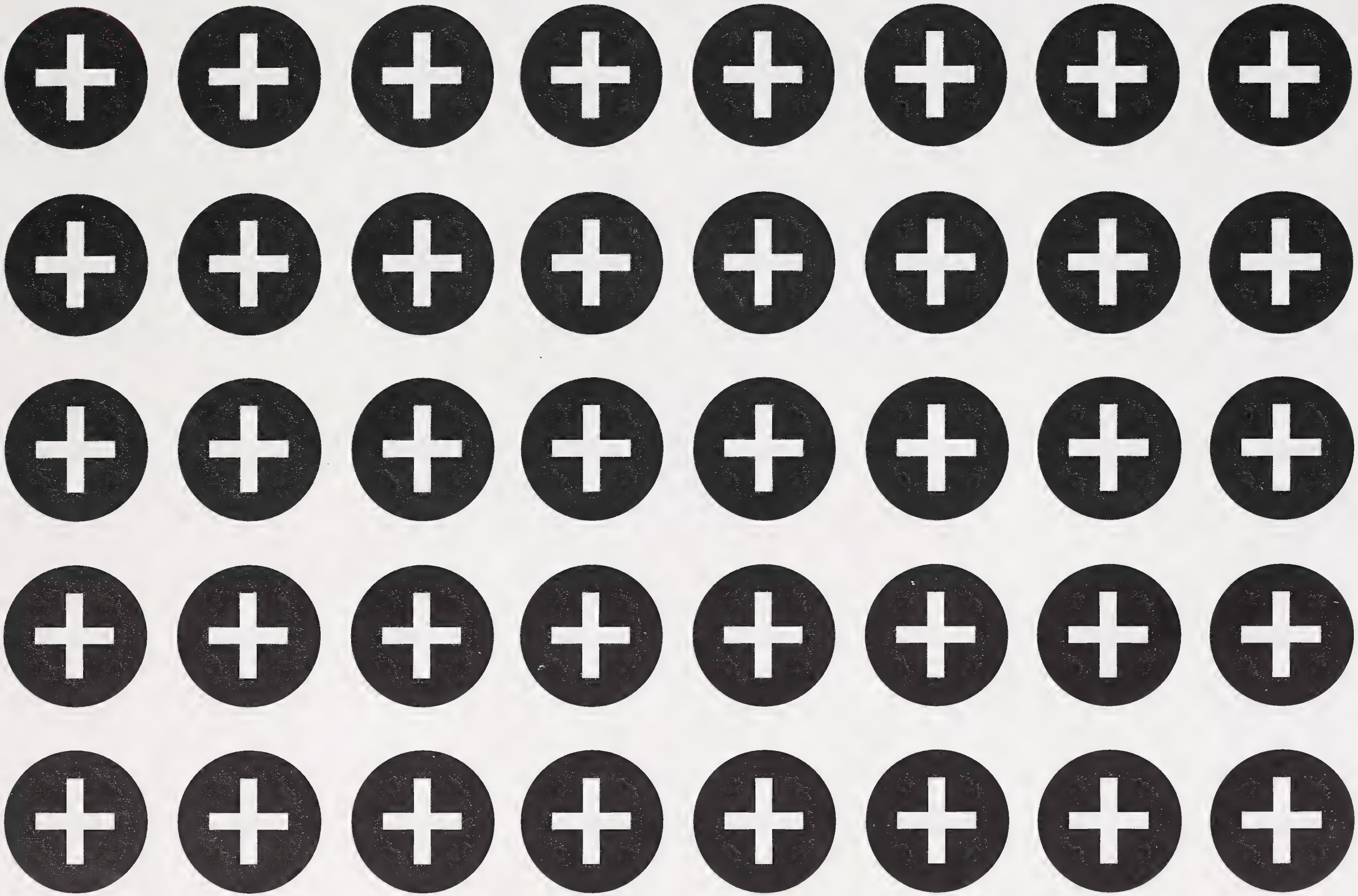
Inequation Scale A



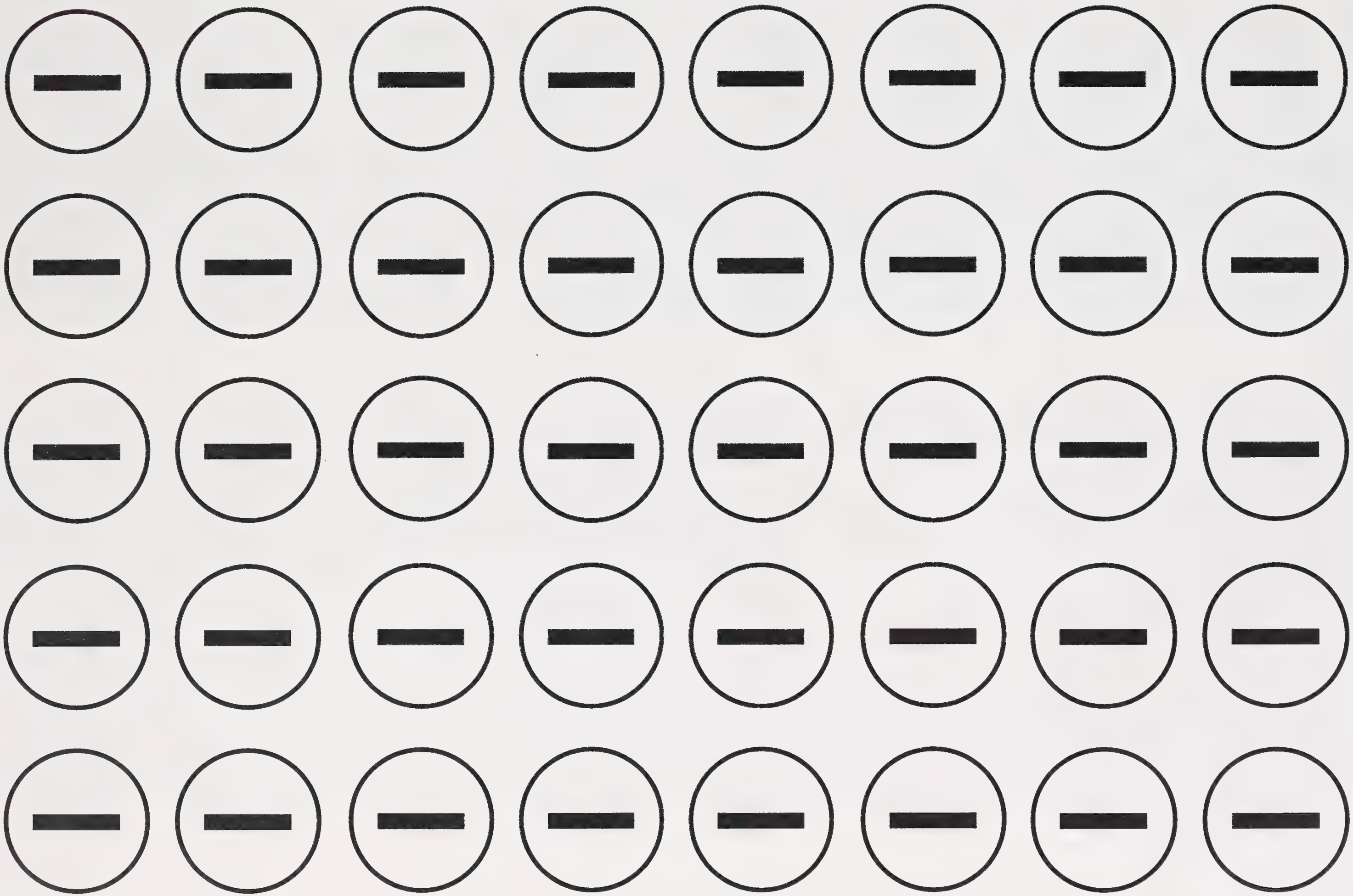
Inequation Scale B



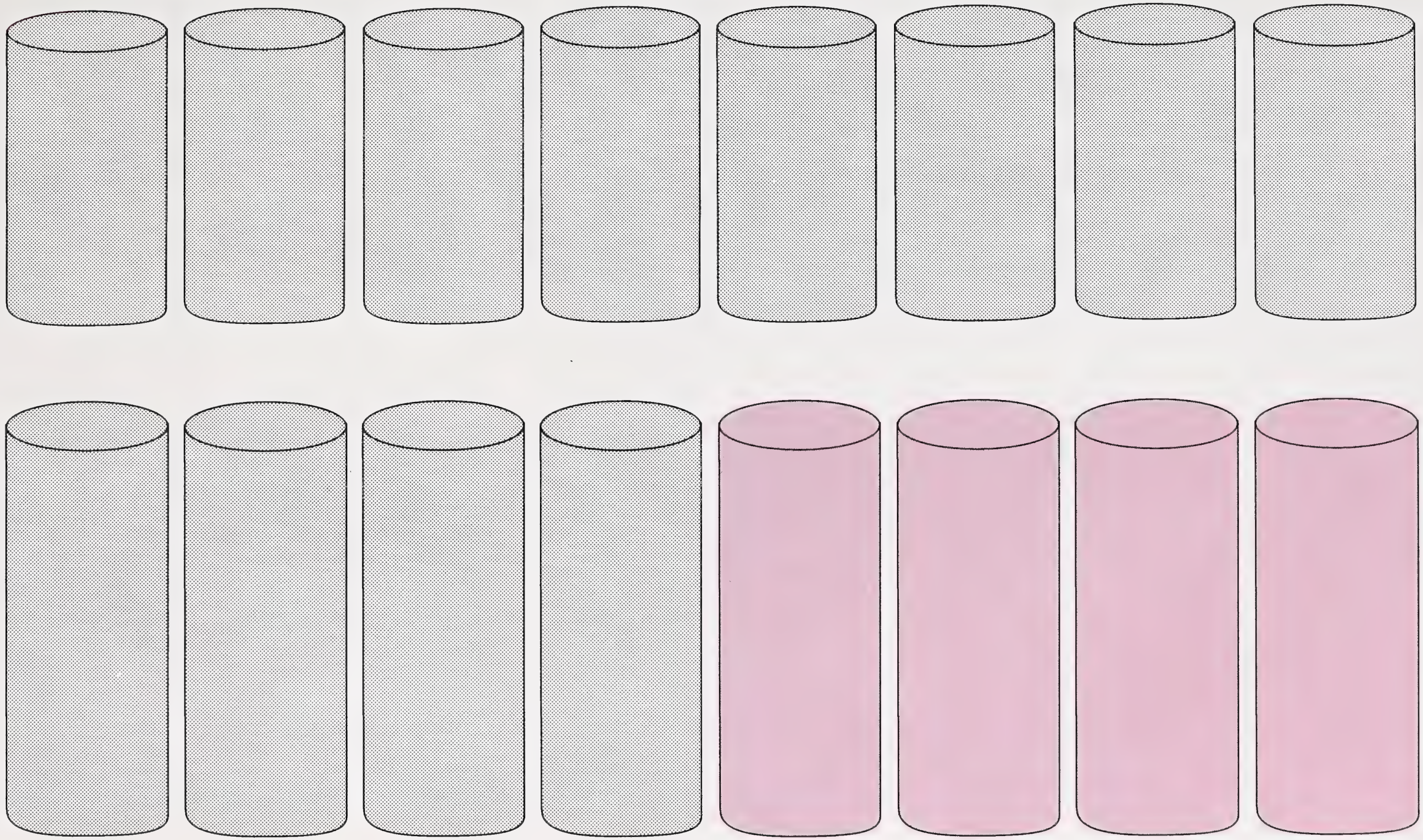
Positive Counters



Negative Counters



Cylinders



NOTES

NOTES

Mathematics 9

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Module 4

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